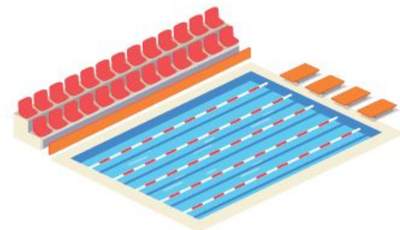
Junior Cycle Sample Assessment Items

Question 1

Sophie is training for a triathlon.
On New Year's Day, Thursday January 1st she went to the pool and swam some lengths.
The next day she began a training programme and swam 7 more lengths than she had on Thursday.



Each day after that, she swam 7 more lengths than the day before. By the following Wednesday night she had swam a total of 161 lengths for the whole week.

(a) How many lengths did Sophie swim on New Year's Day?

Use words, tables/charts and generalised expressions to justify your answer.

Suppose Sophie swam x lengths on January 1st. She swam 7 more on January 2nd and 7 more the next day.

Number of Lengths	**Date**
x	Thu Jan 1st
$x+7$	Fri Jan 2nd
$x+7+7$	Sat Jan 3rd
$x+7+7+7$	Sun Jan 4th
$x+7+7+7+7$	Mon Jan 5th
$x+7+7+7+7+7$	Tue Jan 6th
$x+7+7+7+7+7+7$	Wed Jan 7th

On Jan 2nd she swam x lengths plus 1 set of seven more lengths. On Jan 3rd she swam x lengths plus 2 sets of seven more lengths. If we generalise, on Wed Jan 7th, she will swim x lengths, plus 6 more sets of seven lengths. So we can complete our table as follows:

Number of Lengths	**Date**
x	Thu Jan 1st
$x+7$	Fri Jan 2nd
$x+2(7)$	Sat Jan 3rd
$x+3(7)$	Sun Jan 4th
$x+4(7)$	Mon Jan 5th
$x+5(7)$	Tue Jan 6th
$x+6(7)$	Wed Jan 7th

We know that by Wed Jan 7th, Sophie swam 161 lengths over the preceding week, so let's add up her lengths for the week from the table.

$$x + x + 7 + x + 2(7) + x + 3(7) + x + 4(7) + x + 5(7) + x + 6(7) = 161$$
$$7x + 147 = 161$$
$$7x = 161 - 147$$
$$x = \frac{14}{7}$$
$$x = 2$$

So Sophie swam 2 lengths on January 1st.

We can check our answer by substituting 2 in for x in our first table:

Number of Lengths	**Date**
2	Thu Jan 1st
2 + 7	Fri Jan 2nd
2 + 7 + 7	Sat Jan 3rd
2 + 7 + 7 + 7	Sun Jan 4th
2 + 7 + 7 + 7 + 7	Mon Jan 5th
2 + 7 + 7 + 7 + 7 + 7	Tue Jan 6th
2 + 7 + 7 + 7 + 7 + 7 + 7	Wed Jan 7th

If we add these up we get 161 lengths

$$2 + 9 + 16 + 23 + 30 + 37 + 44 = 161$$

This confirms that Sophie swam 2 lengths on January 1st.

(b) Although it's probably not possible but if Sophie were to continue this pattern, each day swimming 7 more lengths than the day before, on which day would she swim 499 lengths for her daily total?

We need to find the general formula for the number of lengths she swims on day n.

We know Sophie swims two lengths on day 1. Then on day 2 she swims two lengths plus 1 more set of seven lengths. In general, on day n she will swim two lengths plus $n-1$ sets of seven lengths. So on day n Sophie swims $2+(n-1)(7)$ lengths.

Now we can find out after how many days (represented by n) she swims 499 lengths. First let the number of lengths equal to 499:

$$2+(n-1)(7) = 499$$
$$2+7n-7 = 499$$
$$7n = 499-2+7$$
$$n = \frac{504}{7}$$
$$n = 72$$

This means Sophie swims 499 lengths on day 72.

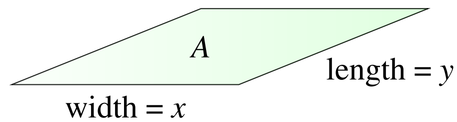
Question 2

A gardener looking at a rectangular flower bed in his garden commented

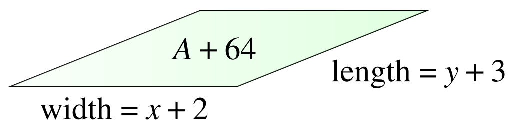
> If I had made that bed $2m$ wider and $3m$ longer it would have been $64m^2$ larger, but if I had made it $3m$ wider and $2m$ longer it would have been $68m^2$ larger. What is the length and width of the garden?

Represent the situation using diagrams and use mathematics to solve the problem.

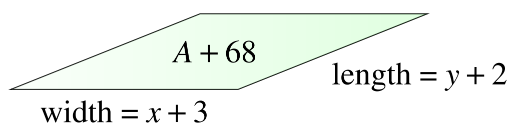
Suppose the width of the flower bed is x and the length is y. Since it is a rectangle, the area is $A = xy$.



The first alternative flower bed is $2m$ wider which means it has width $x + 2$.
It is $3m$ longer which means it has length $y + 3$.
Its area is $64m^2$ bigger which is $A + 64$.



The second alternative flower bed is $3m$ wider which means it has width $x + 3$.
It is $2m$ longer which means it has length $y + 2$.
Its area is $68m^2$ bigger which is $A + 68$.



Now we can form two equations using all of this information:

$$(x+2)(y+3) = A + 64$$
$$(x+3)(y+2) = A + 68$$

To solve the equations, first multiply out the brackets:

$$xy + 3x + 2y + 6 = A + 64$$
$$xy + 2x + 3y + 6 = A + 68$$

Since the area of the original flowerbed $A = xy$, we can say

$$A + 3x + 2y + 6 = A + 64$$
$$A + 2x + 3y + 6 = A + 68$$

Tidying this up we get the simultaneous equations:

$$3x + 2y = 58 \qquad (1)$$
$$2x + 3y = 62 \qquad (2)$$

To solve this, we need to manipulate the equations so the x terms match. Multiply the first equation by 2 and the second equation by 3 to get:

$$6x + 4y = 116$$
$$6x + 9y = 186$$

Now subtract the second equation from the first to get

$$6x - 6x + 4y - 9y = 116 - 186$$
$$\Rightarrow \quad -5y = -70$$
$$\Rightarrow \quad y = \frac{-70}{-5}$$
$$\Rightarrow \quad y = 14$$

So the length of the garden is $14m$. Now use Equation 1 to find the width x:

$$3x + 2y = 58$$
$$\Rightarrow \quad 3x + 2(14) = 58$$
$$\Rightarrow \quad 3x + 28 = 58$$
$$\Rightarrow \quad 3x = 58 - 28$$
$$\Rightarrow \quad 3x = 30$$
$$\Rightarrow \quad x = 10$$

So the flowerbed has width $x = 10m$ and length $y = 14m$. If we substitute these values into the original problem statement we can verify that our solution is correct.

Question 3

Students in a mathematics class made up a secret code where unfamiliar symbols stand for five different mathematical operations ($\times$, $\div$, $+$, $-$, $\hat{}$). Crack the code used in the following equations:

$$(5 \heartsuit 40) \blacklozenge 3 = 15$$
$$(14 \blacklozenge 2 \spadesuit 5) * 2 = 4$$
$$4 \ni (14 \spadesuit 6) \ni 2 = 64$$

Let's take the equations one at a time. In the first equation we can combine 5 and 40 in different ways to make a number in the brackets. Adding them might be useful, because $40+5 = 45$ which can then be divided by 3 to give 15. Let's try this:

$$\begin{aligned} & (5 \heartsuit 40) \blacklozenge 3 = 15 \\ \Rightarrow \quad & (5+40) \div 3 = 15 \\ \Rightarrow \quad & 45 \div 3 = 15 \end{aligned}$$

So far we are guessing the code as follows:

Math Symbol	$+$	$-$	$\times$	$\div$	$\hat{}$
Code Symbol	$\heartsuit$			$\blacklozenge$	

So we have possibly cracked the code for $+$ and $\div$, but we should check the other equations and symbols before we're sure. Assuming that $\blacklozenge$ represents $\div$, the second equation is now:

$$(14 \blacklozenge 2 \spadesuit 5) * 2 = 4$$
$$(14 \div 2 \spadesuit 5) * 2 = 4$$

We are also assuming that $\spadesuit$ does not represent $\hat{}$ since then the terms in brackets would give $(14 \div 2^5)$ which is a very small number. Bearing in mind BOMDAS, we can divide 14 by 2 first to get:

$$(14 \div 2 \spadesuit 5) * 2 = 4$$
$$(7 \spadesuit 5) * 2 = 4$$

We can combine 7 and 5 in a few ways using the symbols we haven't cracked. Those are $-, \times, \hat{}$. Multiplication and powers give large answers, which are probably not what we're looking for, but subtraction does not.

$$(7-5) * 2 = 4$$
$$2 * 2 = 4$$

Be careful here! In this case $*$ could represent $\times$ or $\hat{}$ since $2 \times 2 = 4$ but also $2^2 = 4$.

Now we have

Math Symbol	$+$	$-$	$\times$	$\div$	$\hat{}$
Code Symbol	♥	♠		♦	

Let's use the last equation to figure out the other symbols.
We have two symbols left $\ni, *$ that we need to match with $\times, \hat{}$.

The third equation is

$$4 \ni (14 \spadesuit 6) \ni 2 = 64$$
$$4 \ni (14 - 6) \ni 2 = 64$$
$$4 \ni 8 \ni 2 = 64$$

The power symbol $\hat{}$ doesn't work for $\ni$ here since $4^8 = 65536$ which is too large. But $4 \times 8 \times 2 = 64$ which means the $\ni$ symbol matches with $\times$. Then $*$ must represent the remaining operation $\hat{}$ and thus we have cracked the code as follows:

Math Symbol	$+$	$-$	$\times$	$\div$	$\hat{}$
Code Symbol	♥	♠	$\ni$	♦	$*$

Based on this, the three equations are:

$$(5 + 40) \div 3 = 15$$
$$(14 \div 2 - 5)\hat{}2 = 4$$
$$4 \times (14 - 6) \times 2 = 64$$

If we evaluate these expressions by hand or using a calculator, we can see that they are correct. So we have correctly cracked the code.

The students results will be verified by asking them to evaluate this expression:

$$2 * 3 \diamondsuit (4 \ni 3 \spadesuit 10) \heartsuit 6 * 2 \ni (4 \heartsuit 1) \diamondsuit (1 * 4 \ni 20)$$

What answer should the group get? Justify your answer and use words to explain your thinking.

Using the code we cracked above, this equation becomes

$$2\hat{}3 \div (4 \times 3 - 10) + 6\hat{}2 \times (4 + 1) \div (1\hat{}4 \times 20)$$

We should evaluate the expressions in brackets first, followed by the powers:

$$\begin{aligned} & 2\hat{}3 \div (4 \times 3 - 10) + 6\hat{}2 \times (4 + 1) \div (1\hat{}4 \times 20) \\ & = 2\hat{}3 \div (2) + 6\hat{}2 \times (5) \div (20) \\ & = 8 \div 2 + 36 \times 5 \div 20 \end{aligned}$$

Now perform the multiplication and division before adding or subtracting:

$$\begin{aligned} & 8 \div 2 + 36 \times 5 \div 20 \\ & = 4 + 9 \\ & = 13 \end{aligned}$$

So the group should get 13 as the answer if they correctly cracked the code.

Question 4

Fully factorise the following polynomials

$$12x^2 - 27 \qquad 18x^2 - 60x + 50 \qquad 12x^2 + 23x - 24$$

(a) To factorise $12x^2 - 27$ notice that both terms have a common factor of 3 so we can write

$$12x^2 - 27 = (3)(4x^2) - (3)(9) = 3(4x^2 - 9)$$

Although there are no more common factors, the terms in brackets can be written as the difference of two squares as follows:

$$3(4x^2 - 9) = 3((2x)^2 - 3^2) = 3(2x + 3)(2x - 3)$$

using the rule $(a^2 - b^2) = (a + b)(a - b)$.

(b) To factorise $18x^2 - 60x + 50$, notice that there is a common factor of 2 in every term so we can write $18x^2 - 60x + 50 = 2(9x^2 - 30x + 25)$.

To factorise $9x^2 - 30x + 25$ we need to write

$$9x^2 - 30x + 25 = (3x + a)(3x + b)$$

Let's list some possible factors and calculate their x term:

Factors	**x-term**
$(3x - 1)(3x - 25)$	$(3x)(-25) + (-1)(3x) = -76x$
$(3x - 25)(3x - 1)$	$(3x)(-1) + (-25)(3x) = -78x$
$(3x - 5)(3x - 5)$	$(3x)(-5) + (-5)(3x) = -30x$

So the last pair of factors $(3x - 5)(3x - 5)$ will work. We can verify the factorisation by multiplying the two factors:

$$\begin{aligned}(3x - 5)(3x - 5) &= 9x^2 - 15x - 15x + 25 \\ &= 9x^2 - 30x + 25\end{aligned}$$

Finally, we can say that the full factorisation of the original equation is:

$$\begin{aligned}18x^2 - 60x + 50 &= 2(9x^2 - 30x + 25) \\ &= 2(3x - 5)(3x - 5)\end{aligned}$$

(c) To factorise $12x^2 + 23x - 24$ we write $12 = 3 \times 4$ and list some possible factors of $12x^2 + 23x - 24$ in order to calculate their x term:

Factors	***x*-term**
$(4x-1)(3x+24)$	$(4x)(24)+(-1)(3x) = 93x$
$(4x-24)(3x+1)$	$(4x)(1)+(-24)(3x) = -68x$
$(4x-2)(3x+12)$	$(4x)(12)+(-2)(3x) = 42x$
$(4x-12)(3x+2)$	$(4x)(2)+(-12)(3x) = -28x$
$(4x-3)(3x+8)$	$(4x)(8)+(-3)(3x) = 23x$
$(4x-8)(3x+3)$	$(4x)(3)+(-8)(3x) = -12x$
$(4x-4)(3x+6)$	$(4x)(6)+(-4)(3x) = 12x$
$(4x-6)(3x+4)$	$(4x)(4)+(-6)(3x) = -2x$

So the pair of factors $(4x-3)(3x+8)$ will work. We can verify the factorisation by multiplying the two factors:

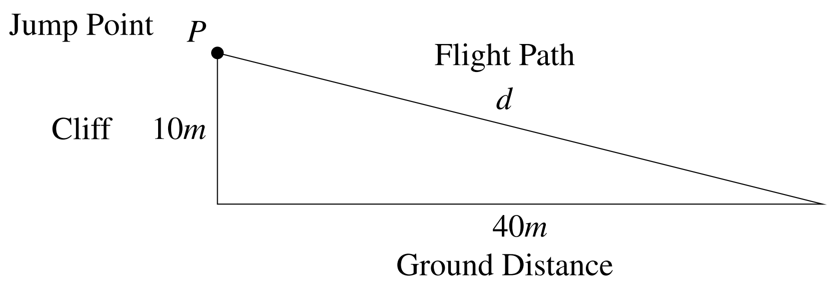
$$\begin{aligned}(4x-3)(3x+8) &= 12x^2 + 32x - 9x - 24 \\ &= 12x^2 + 23x - 24\end{aligned}$$

Finally, we can say that the full factorisation of the original equation is:

$$12x^2 + 23x - 24 = (4x-3)(3x+8)$$

Question 5

Marianna enjoys paragliding.
She decides to make her first jump from a 10m cliff.
She glides along a straight line, covering 40m of ground.
Represent this information in a diagram. Label the cliff, the point from where Marianna takes her jump, the ground distance and the flight path. What assumptions have you made?

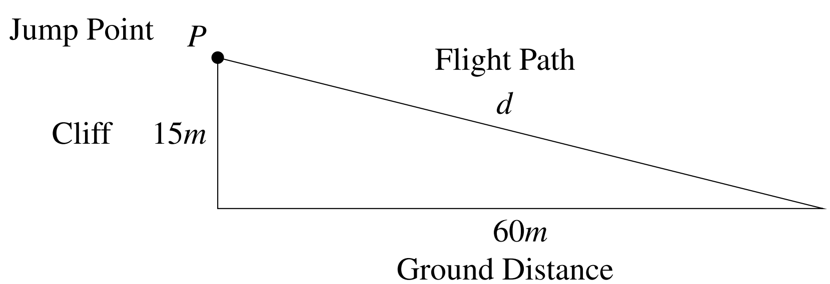


We have assumed that Marianna falls at a constant speed in a straight line from the jump point without experiencing any wind resistance or other forces that would change her flight path.

(a) After several successful flights, she decides to go to a higher cliff. The cliff is 15m high. How much ground distance does the glider cover from the higher cliff? *Assume the steepness of the flight path remains the same.*

We need to figure out the steepness of the path from her first jump. This is the slope of the straight line that represents the flight path. The rise of this line is $-10m$ (10m downwards) while the run is 40m. Since the slope (steepness) doesn't change, we can say that for every x metres Marianne goes downwards, she moves $4x$ metres horizontally along the ground.

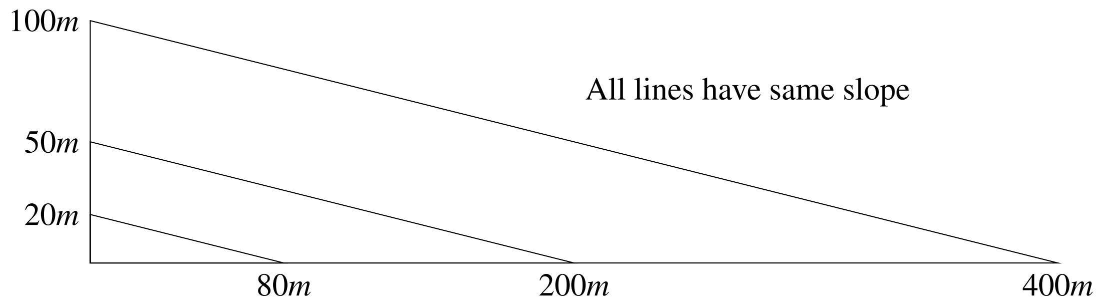
So a drop of 15m will mean she covers $4 \times 15 = 60$ metres ground distance. Marianna finishes her flight after covering 60m of ground.



(b) Marianna makes flights from three cliffs that are $20m$, $50m$ and $100m$ high. How much ground distance does the glider cover on each flight?

Since the steepness of the flight path (the slope) remains the same for each jump, the ground covered is always 4 times bigger than the distance fallen. Thus we can use multiplication to say that:

Height of Jump	**Ground Distance Travelled**
$20m$	$20 \times 4 = 80m$
$50m$	$50 \times 4 = 200m$
$100m$	$100 \times 4 = 400m$



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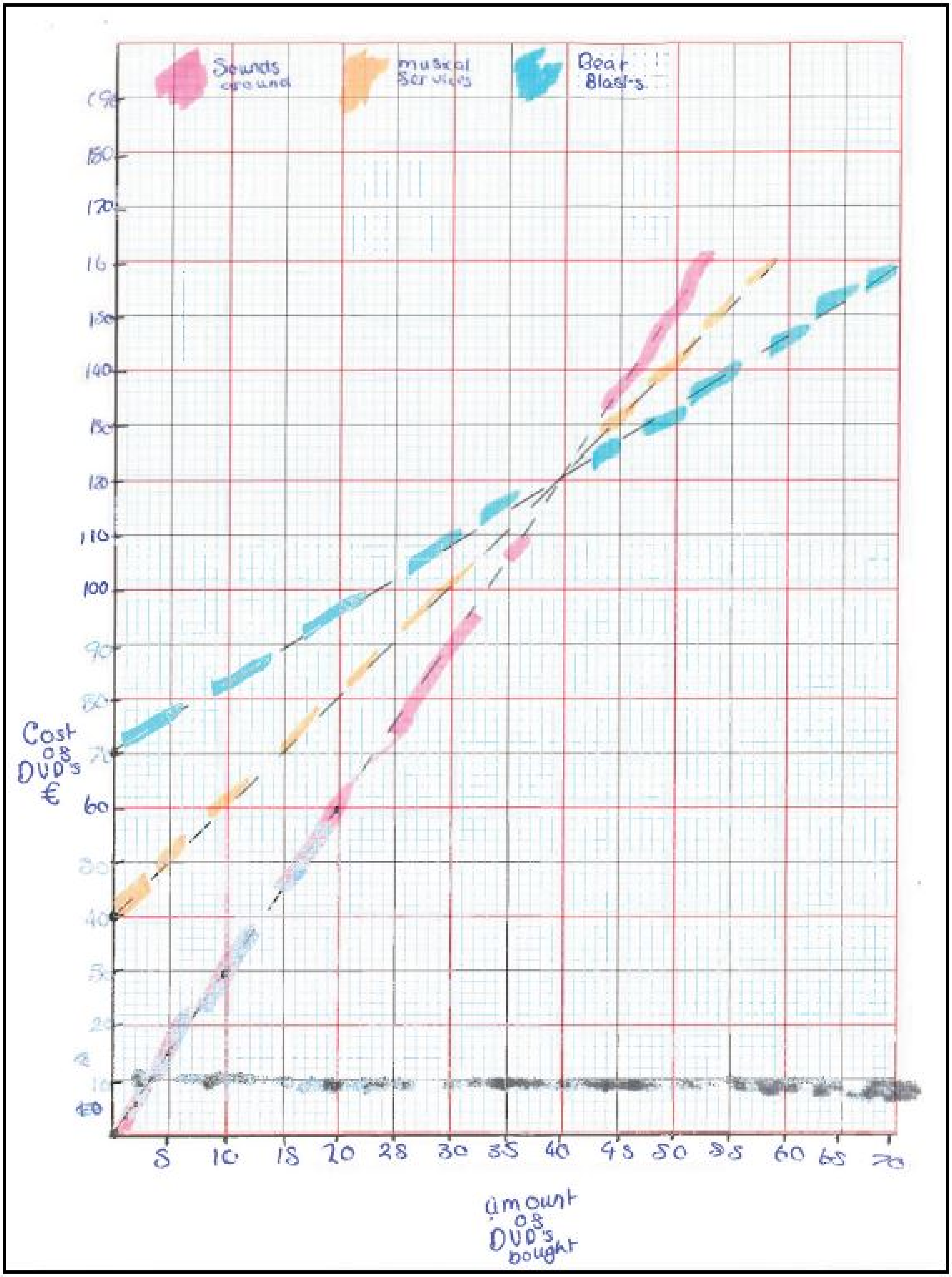
Use your representation, other helpful diagrams and mathematics to justify your solution in each situation.

Question 6

Lucy was investigating the cost of DVDs from three different suppliers Sounds Around, Musical Services and Beat Blasts. She represented the information in the graph below.



Interpret the graph to help you decide which company offers the best value justify your answer with words and mathematics.

How much would 1000 DVDs cost in BeatBlasts?

The graph does not show pricing for 1000 DVDs. We can see that the cost of DVDs in BeatBlasts is represented by a straight line. If we can work out the equation of the line, we can use that to work out the cost of any amount of DVDs.

BeatBlasts is represented by the blue line. Let's pick two points on the line to estimate the slope, $(0, 70)$ and $(40, 120)$. When moving from $(0, 70)$ to $(40, 120)$ the line rises 50 units and moves 40 units horizontally. Therefore the slope is given by

$$slope = \frac{rise}{run} = \frac{50}{40} = \frac{5}{4}$$

Now use the point $(0, 70)$ and the slope $\frac{5}{4}$ to find the equation of the line as follows:

$$\begin{aligned} y - 70 &= \frac{5}{4}(x - 0) \\ y - 70 &= \frac{5x}{4} \\ 4y - 280 &= 5x \end{aligned}$$

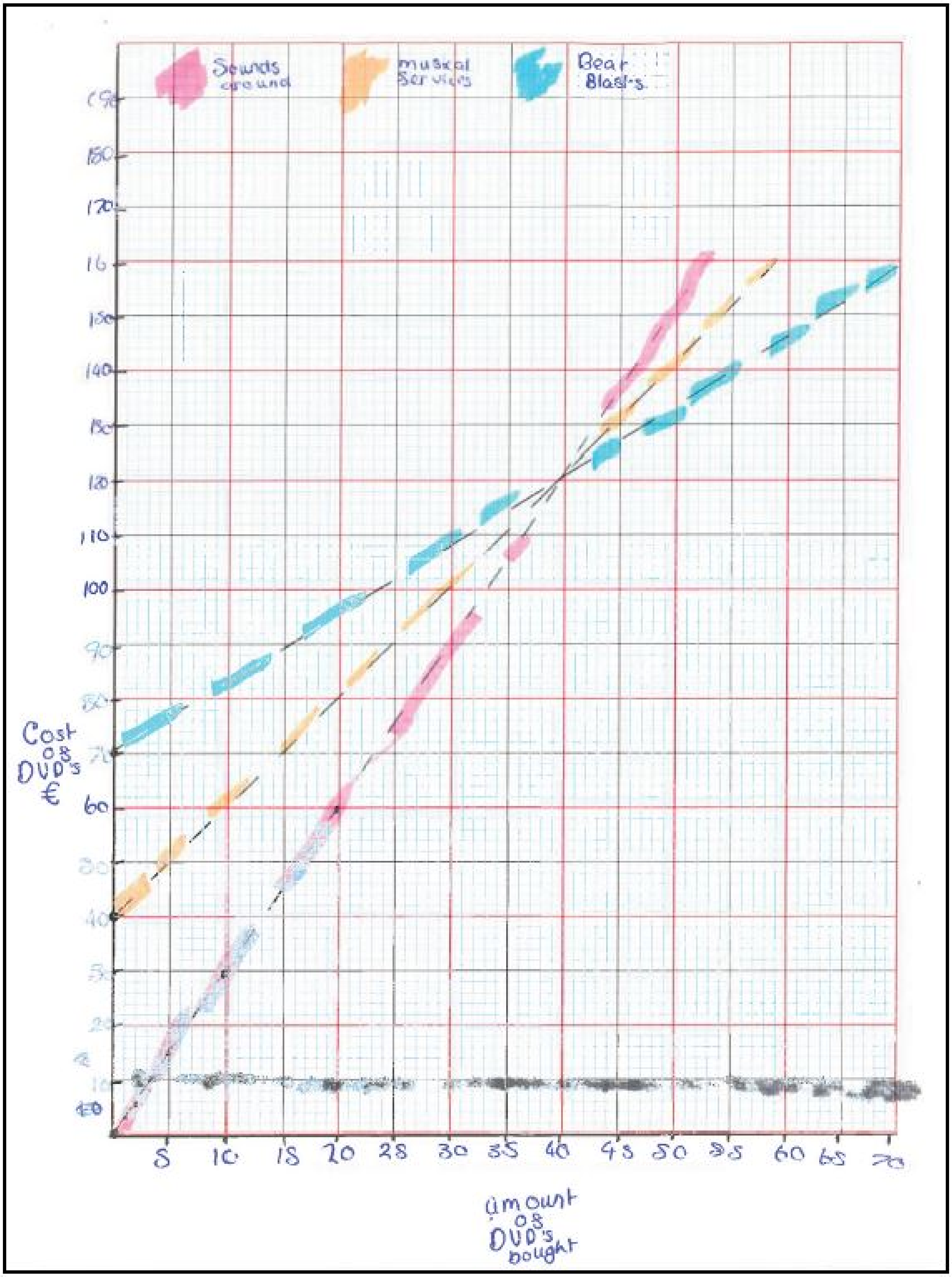
Now we can substitute $x = 1000$ to find the cost y:

$$\begin{aligned} 4y - 280 &= 5(1000) \\ 4y &= 5000 - 280 \\ y &= \frac{4720}{4} \\ y &= 1180 \end{aligned}$$

So 1000 DVDs would cost €1180 from BeatBlasts.

Question 7

Lucy was investigating the cost of DVDs from three different suppliers: Sounds Around, Musical Services and Beat Blasts. She represented the information in the graph below.



Interpret the graph to help you decide which company offers the best value, Justify your answer with words and mathematics.

How much would you pay for 20 DVDs in Sounds Around?

Sounds Around is represented by the red line. Find 20 on the horizontal axis and follow vertically upwards to the point $(20, 60)$ on the red line. This indicates that 20 DVDs cost €60 in Sounds Around.

If you had to buy 25 DVDs where would you buy them? Why would you buy them there?

Draw a line vertically upwards from 25 on the horizontal axis and you'll see that the red line is the lowest cost, the blue line is the highest and the yellow line is in between.

I would buy 25 DVDs from Sounds Around since they are the cheapest at approximately €72. The other two companies charge more.

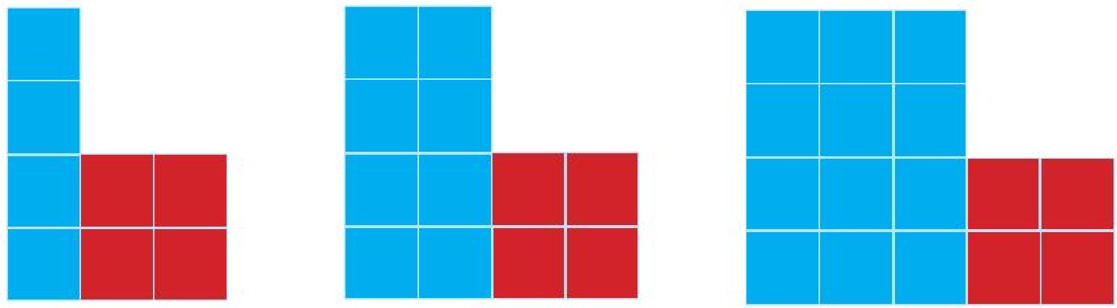
If you had to buy 100 DVDS where would you buy them? Give a reason for your answer.

Looking at the lines, we can see that the red line has the steepest slope. We can see it will be more expensive for all amounts above 40 DVDs. The blue line has the lowest slope so we expect 100 DVDs will be cheaper from BeatBlasts.

So I would buy 100 DVDs from BeatBlasts.

Question 8

Students in a mathematics class make a sequence of shapes using red and blue tiles.



Shape Number 1 Shape Number 2 Shape Number 3

What is the total number of tiles in Shape number n?

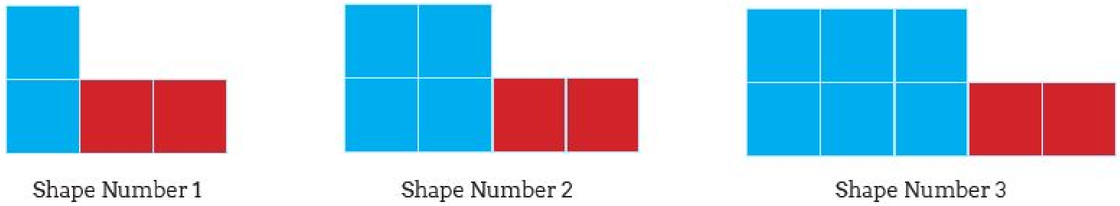
Each new shape has the same number of red tiles, but the number of blue tiles increases by 4 each time. We can represent this in a table:

Shape Number	**Number of tiles**
1	4 red + 4 blue
2	4 red + (4+4) blue
3	4 red + (4+4+4) blue
⋮	

If we count up we can see that the tenth shape will have 4 red tiles plus 10 sets of four blue tiles. If we generalise, we can say that the nth shape has 4 red tiles plus n sets of four blue tiles. The table looks like this:

Shape Number	**Number of tiles**
1	4 red + 4 blue
2	4 red + 2(4) blue
3	4 red + 3(4) blue
⋮	
10	4 red + 10(4) blue
⋮	
n	4 red + $n(4)$ blue

The total number of tiles in shape $4 + n(4) = 4n + 4 = 4(n + 1)$.



What is the total number of tiles in Shape number n now?

In this pattern each new shape has the same number of red tiles, but the number of blue tiles increases by 2 each time. We can represent this in a table:

Shape Number	**Number of tiles**
1	2 red + 2 blue
2	2 red + (2+2) blue
3	2 red + (2+2+2) blue
⋮	

If we count up we can see that the tenth shape will have 2 red tiles plus 10 sets of two blue tiles. If we generalise, we can say that the nth shape has 4 red tiles plus n sets of four blue tiles. The table looks like this:

Shape Number	**Number of tiles**
1	2 red + 2 blue
2	2 red + 2(2) blue
3	2 red + 3(2) blue
⋮	
10	2 red + 10(2) blue
⋮	
n	2 red + $n(2)$ blue

The total number of tiles in shape n is $2n + 2 = 2(n + 1)$. This is exactly half the number of tiles in the first pattern.

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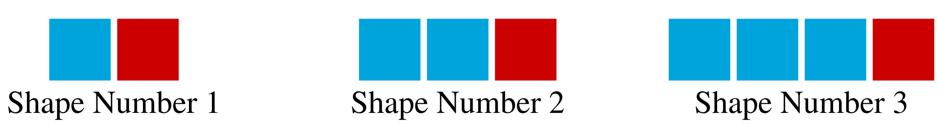
If I remove half the tiles again what is the number of tiles in shape n? What would the tile sequence look like now?

The first pattern had $4(n + 1)$ tiles, the second had $2(n + 1)$ tiles. So if we divide by 2 again the pattern would have $n + 1$ tiles in shape n. It would look like this:



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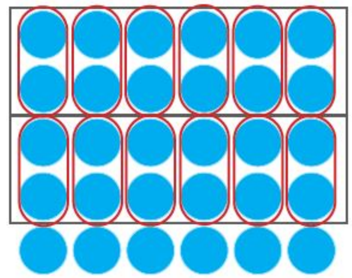
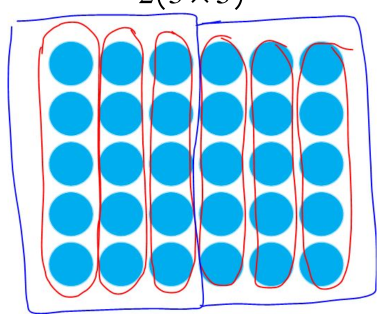
Question 9

Use the diagrams below to illustrate the mathematical expressions. The first one has been done for you.

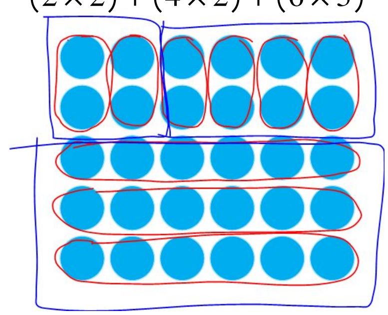
$2(6\times2)+6$



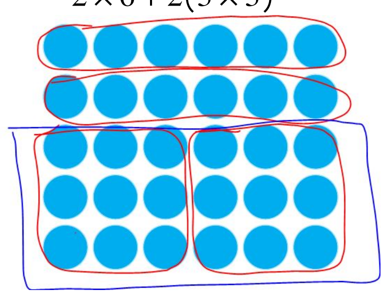
$2(3\times5)$



$(2\times2)+(4\times2)+(6\times3)$



$2\times6+2(3\times3)$



Question 10

In a school lessons are 55 minutes long.

A mathematics lesson starts at 9:15am. What time does the lesson end?

We can count forward 45 minutes to 10am. The lesson last 10 minutes more than that (since 45+10=55 minutes) so it finishes at 10:10am.

A PE lesson ends at 3:30pm. At what time does the lesson start?

We can count backwards 30 minutes to 3pm. The lesson started 25 minutes before that (since 30+25=55 minutes) so it started at 2:35pm.

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Lunch break is 1 $\frac{1}{4}$ hours long. Lunch break ends at 1:30pm. At what time does is start?

$\frac{1}{4}$ of an hour is $\frac{1}{4} \times 60 = 15$ minutes. We can count back one hour to 12:30pm, but the lunch break lasts 15 minutes more than that. So count back another 15 minutes to 12:15pm. Lunch started at 12:15pm.

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Question 11

Work out the answer.

Make sure to perform brackets first, multiplication and division next and addition and subtraction last (BOMDAS).

$$3 + (24 \div 3) + 5 = 3 + (8) + 5 = 16$$

Put brackets in the calculation below to make it correct

$$3 + 24 \div 3 + 5 = 6$$

There are a few choices, so let's list some of them and see can we find one that is correct:

$(3 + 24) \div 3 + 5$	$3 + (24 \div 3) + 5$	$3 + 24 \div (3 + 5)$
$= 27 \div 3 + 5$	$= 3 + 8 + 5$	$= 3 + 24 \div 8$
$= 9 + 5$	$= 16$	$= 3 + 3$
$= 14$	$= 16$	$= 6$

So putting brackets around the last two terms works since $3 + 24 \div (3 + 5) = 6$.

When $y = 1$, which expression below has the largest value?

$$3 + y \qquad 10 - y \qquad y^2 \qquad 3y \qquad \frac{y}{2}$$

Substitute $y = 1$ into each expression to get:

$3 + 1$	$10 - 1$	1^2	$3(1)$	$\frac{1}{2}$
$= 4$	$= 9$	$= 1$	$= 3$	$= \frac{1}{2}$

So the second expression $10 - y$ has the largest value when $y = 1$.

When $y = 4$, which expression below has the largest value?

$$3 + y \qquad 10 - y \qquad y^2 \qquad 3y \qquad \frac{y}{2}$$

Substitute $y = 4$ into each expression to get:

$3 + 4$	$10 - 4$	4^2	$3(4)$	$\frac{4}{2}$
$= 7$	$= 6$	$= 16$	$= 12$	$= 2$

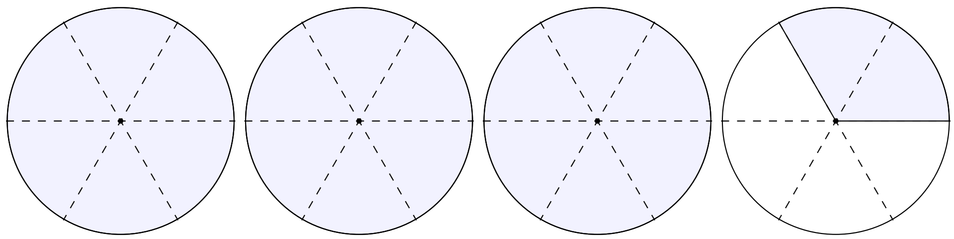
So the third expression y^2 has the largest value when $y = 4$.

Question 12

How many sixths are there in $3\frac{1}{3}$?

Justify your answer with a diagram.

Start by drawing 4 circles. We'll fill the first three to represent the whole number part. Then split the last circle into three parts and shade one of them.

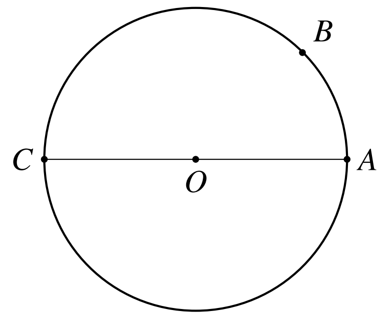


If we add some lines to split the circle into 6 equal parts we can count them up. There are $6 + 6 + 6 + 2 = 20$ sixths in $3\frac{1}{3}$.

Question 13

A, B and C, are points on a circle, centre O. AC is a diameter of the circle.

Represent this information in a diagram.



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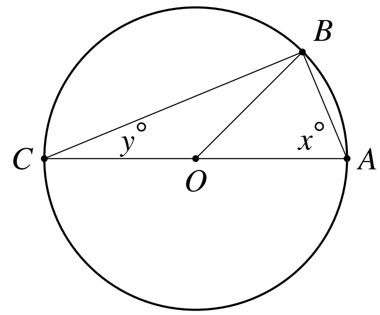
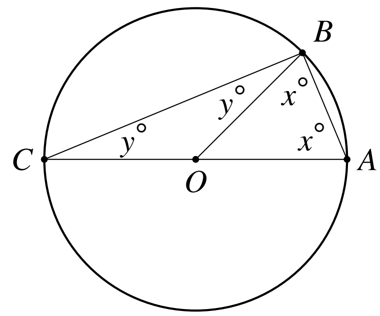
Angle BAO is x° and angle BCO is y°. Mark this information on your diagram.
Explain why angle ABO must be x° and angle CBO must be y°.

Use algebra to show that angle ABC must be 90°.



Look at the triangle ABO in the diagram above. The sides OB and OA are both radii of the circle, so they are the same length. This means the triangle ABO is an isosceles triangle and the angles $\angle ABO$ and $\angle BAO$ must both be x°.

The triangle CBO in the diagram above. The sides OB and OC are both radii of the circle, so they are the same length. This means the triangle CBO is an isosceles triangle and the angles $\angle CBO$ and $\angle BCO$ must both be y°.



We can see that the triangle ABC has three angles of size y°, x° and $x^\circ + y^\circ$ respectively. Since the angles in any triangle always sum to 180° we can say that

$$\begin{aligned}\angle ACB + \angle CAB + \angle ABC &= 180^\circ\\ y^\circ + x^\circ + (x^\circ + y^\circ) &= 180^\circ\\ 2x^\circ + 2y^\circ &= 180^\circ\\ x^\circ + y^\circ &= 90^\circ\end{aligned}$$

This means the angle $\angle ABC = x^\circ + y^\circ = 90^\circ$.

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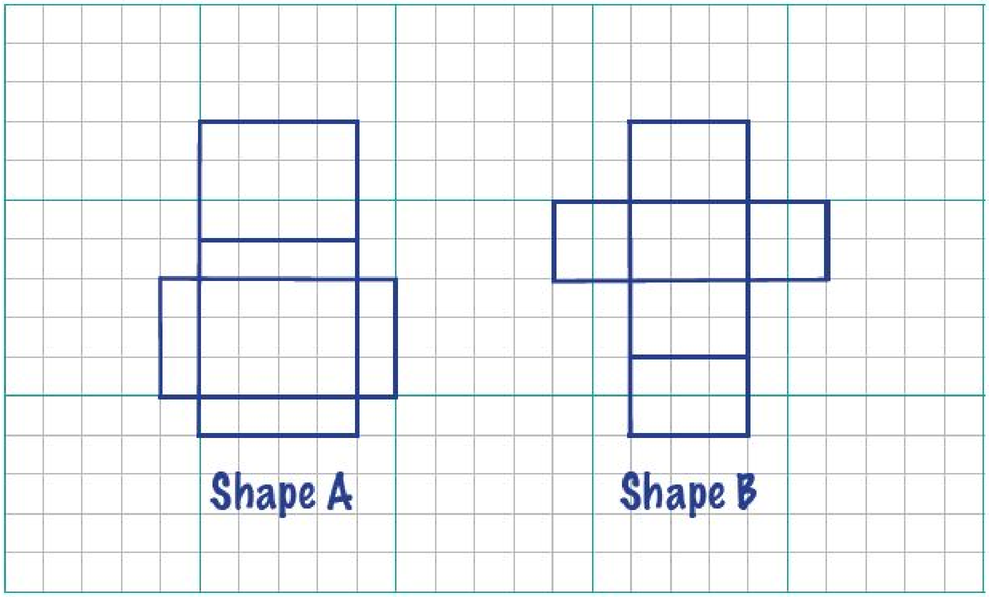
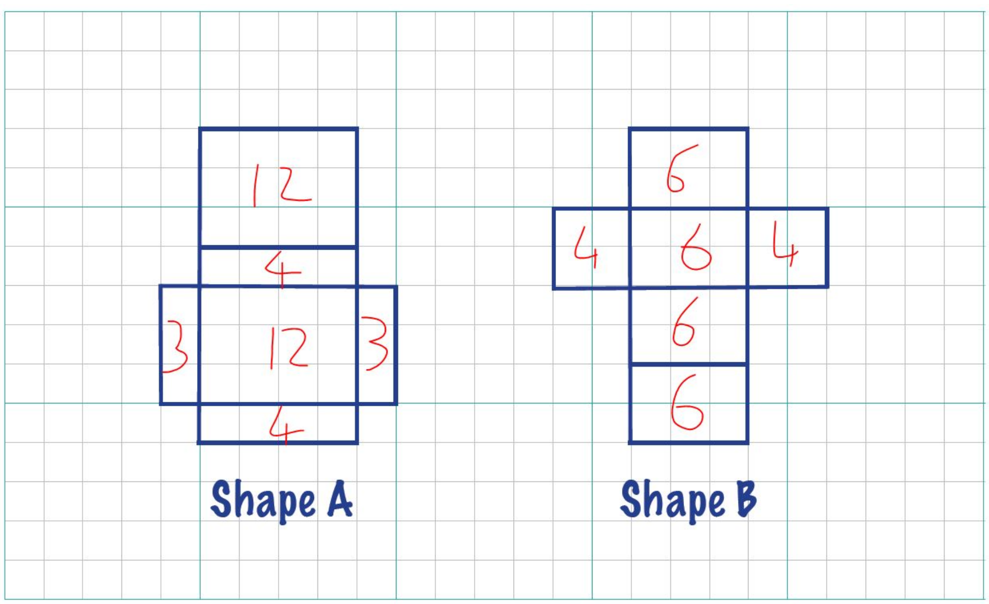
Question 14

The diagram shows two cuboids labelled shape *A* and Shape *B*.



Do the cuboids have the same surface area? Show calculations to show how you know.

Find the surface area of each rectangular part of the cuboid by multiplying length by width or simply by counting the squares in each rectangular section.



The surface area of Shape *A* is $12 + 4 + 3 + 12 + 3 + 4 = 38$ units squared. The surface area of Shape *B* is $6 + 4 + 6 + 4 + 6 + 6 = 32$ units squared. So Shape *A* has a larger surface area than Shape B.

Do the cuboids have the same volume?

Show calculations to show how you know.

When Shape A is folded, it forms a box of length 4, height 1 and width 3. So its volume is $4 \times 1 \times 3 = 12$ units cubed.

When Shape B is folded, it forms a box of length 3, height 2 and width 2. So its volume is $3 \times 2 \times 2 = 12$ units cubed.

So both cuboids have the same volume.

Question 15

A cup of coffee costs €3.20.

The table shows how much different people receive from the sale of a cup of coffee.

Growers	€0.09
Retailers	€0.78
Others	€2.33

Use mathematics to work out what percentage of the cost of a cup of coffee goes to retailers, growers and others.

We know the total cost of a cup of coffee is €3.20. The growers portion is €0.09. To find what percentage this is we calculate as follows:

$$\frac{€0.09}{€3.20} \times 100 = 2.8125\% \text{ or } 3\% \text{ to the nearest percentage point.}$$

Retailers make €0.78 so their percentage is

$$\frac{€0.78}{€3.20} \times 100 = 24.375\% \text{ or } 24\% \text{ to the nearest percentage point.}$$

Finally, Others make €2.33 so their percentage is

$$\frac{€2.33}{€3.20} \times 100 = 72.8125\% \text{ or } 73\% \text{ to the nearest percentage point.}$$

Complete the table with your answers

Growers	3%
Retailers	24%
Others	73%

Some people think that growers should get more. Suppose the percentages changed to

Growers	10%
Retailers	23%
Others	67%

If the retailers still get €0.78 from the sale of a cup of coffee. Use mathematics to work out how much would a cup of coffee cost?

We know that the retailers make €0.78 and this is 23%. Let's find 1% by dividing by 23 first:

$$€0.78 = 23\%$$
$$\frac{€0.78}{23} = 1\%$$
$$€0.0339 = 1\%$$

Now multiply both sides by 100 to find 100%:

$$€0.0339 = 1\%$$
$$\Rightarrow \quad €3.39 = 100\%$$

So a cup of coffee would cost €3.39.

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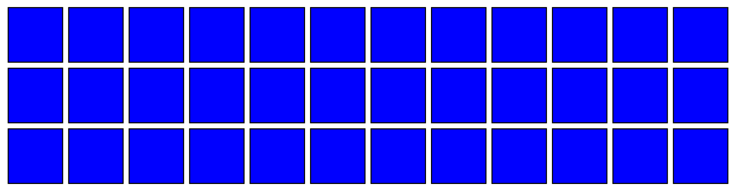
Question 16

Represent 36 in 8 different ways

- As an Array: here is 36 represented as a 12×3 array.



- Using at least one fraction and a multiply symbol

$$36 = \frac{1}{2} \times 72$$

- In a way that shows it is an even number

$$36 = 18 \times 2$$

- In a way that shows it is a multiple of 3

$$36 = 12 \times 3$$

- Using 2 "×" symbols

$$36 = 4 \times 3 \times 3$$

- In a way that shows it is a square number

$$36 = 6^2$$

- Using a power

$$36 = 3^2 \times 4$$

- Using 2 "×" symbols, brackets and a "+" symbol

$$36 = 2 \times 3 \times 4 + 12$$

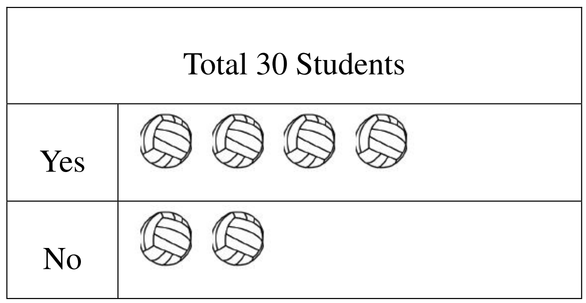
Question 17

Sean asked 30 students if they played GAA.
20 students said yes. 10 students said no.

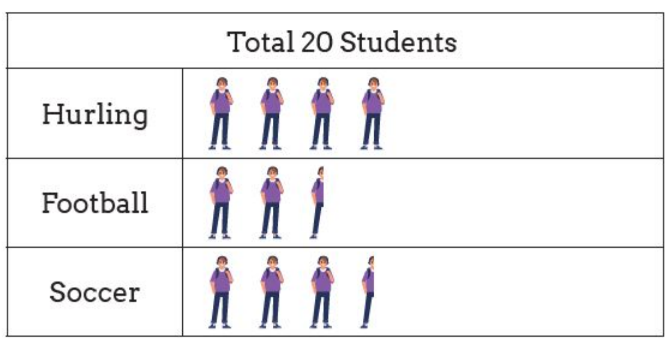
He started to put this information in a table using the key represents 5 students.

Complete the table to show Sean's results.

Since the four footballs in the yes row represent 20 people, each football represents $\frac{20}{4} = 5$ people. So the 'No' row should have two footballs to represent 10 people.



Sarah asked 20 students which sport they like best. She put this information in a table but forgot to write the key.



How many students does represent?

First we need to find how many images of students are used in the table. There are $4 + 2.5 + 3.5 = 10$ images in the table. These 10 images represent 20 students so each image represents $\frac{20}{10} = 2$ students. So the table represents $4 \times 2 = 8$ students who like hurling best, $2.5 \times 2 = 5$ who like football and $3.5 \times 2 = 7$ who like soccer. Just to check: $8 + 5 + 7 = 20$ students.

Question 18

The number chain below is part of a doubling number chain. Fill in the two missing numbers.

The final number is 480 since $2 \times 240 = 480$.

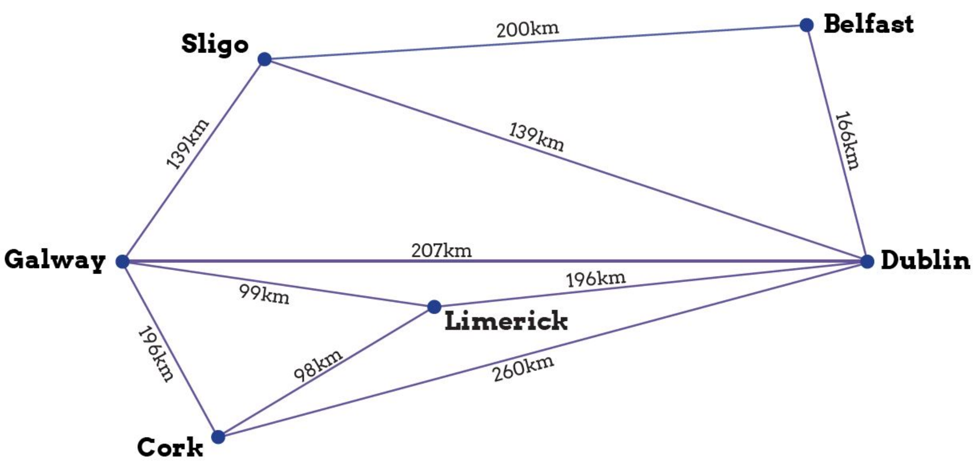
For the first number, we can work backwards along the doubling chain by halving each number and since $\frac{1}{2} \times 60 = 30$m the first number is 30.

The number chain below is part of a halving number chain. Fill in the two missing numbers.

We need to multiply by $\frac{1}{2}$ each time so $\frac{1}{2} \times 10 = 5$ which is the first missing number. Then $\frac{1}{2} \times 5 = 2.5$ or $\frac{5}{2}$ which is the second missing number.

Question 19

Look at this diagram. It shows distances in kilometres between some cities.



(a) How far is it from Dublin to Cork?

(b) Which two cities are 98km apart?

> (a) It is 260km from Dublin to Cork.
>
> (b) Cork and Limerick are 98km apart.
>
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Caoimhe lives in Limerick. She wants to visit either Dublin or Cork. Which of these cities is nearer to Limerick? Tick (✓) your answer.

☐ Dublin ☑ Cork

How many km nearer to Limerick is it?

> Limerick is 196km from Dublin and 98km from Cork, so Cork is closer. The difference between travelling to Dublin and travelling to Cork is $196 - 98 = 98km$. So Cork is 98km nearer to Limerick.
>
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Conor drives from Dublin to Galway to Limerick and back again to Dublin. How many *km* does he drive altogether? Show how you worked out your answer.

Dublin to Galway is $207km$.
Galway to Limerick is $99km$.
Limerick to Dublin to $196km$.

We add up the three distances to see that Conor drives $207 + 99 + 196 = 502km$.

Question 20

Look at these three number cards.

0	8	6

You can put them together to show different numbers. For example: Eighty Six

0	8	6

Put the three cards together in a different way. Write in words what number the cards show.

6	0	8

The new number is six hundred and eight.

Now put the three cards together in another different way. Write in words what number the cards show.

6	8	0

The new number is six hundred and eighty.

Here are three different number cards.

6	9	4

What is the biggest number you can show with these cards?

To make the biggest number, let's put the largest digit in the hundreds position to make as many hundreds as possible, then the next largest in the tens position, and the smallest number in the units position.

9	6	4

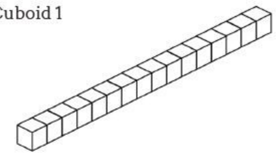
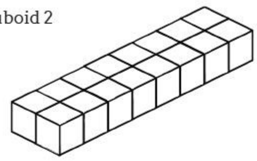
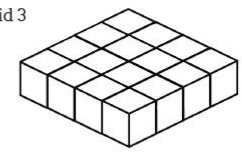
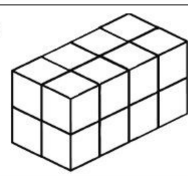
What is the biggest even number you can show with these cards?

To make an even number the last digit must be either 4 or 6. If we choose 4 for the last digit, we can use the bigger number 6 in the tens position, and finally 9 in the hundreds position. So the biggest even number is also

9	6	4

Question 21

You can make only four different cuboids with 16 cubes. Complete the table below showing the dimensions of each of the cuboids that can be made.

	Dimensions		
Cuboid 1 	1	1	16
Cuboid 2 	1	2	8
Cuboid 3 	1	4	4
Cuboid 4 	2	2	4

Which of the cuboids 1 and 4 has the larger surface area?
Use words, diagrams and /or numbers to explain how you know.

Let's look at the six faces of Cuboid 1. Two have area $1 \times 1 = 1$, two have area $1 \times 16 = 16$ and two more have area $1 \times 16 = 16$. So the total surface area is

$$\begin{aligned} &2 \times 1 + 2 \times 16 + 2 \times 16 \\ &= 2 + 32 + 32 \\ &= 66 \text{ units cubed} \end{aligned}$$

Let's look at the six faces of Cuboid 4. Two have area $2 \times 2 = 4$, two have area $2 \times 4 = 8$ and two more have area $4 \times 2 = 8$. So the total surface area is

$$\begin{aligned} &2 \times 4 + 2 \times 8 + 2 \times 8 \\ &= 8 + 16 + 16 \\ &= 40 \text{ units cubed} \end{aligned}$$

So Cuboid 1 has a greater surface area than Cuboid 4.

Write a generalised expression you could use to work out the surface area of any cuboid.

Every cuboid has a length l, width w and height h. As we calculated above, there are six faces. Two of them have area $l \times w$, two have area $l \times h$ and two have area $w \times h$.

So the total surface area is

$$2lw + 2lh + 2wh$$
$$= 2(lw + lh + wh)$$

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Which of the cuboids has the larger volume? Use words, diagrams and /or numbers to explain how you know.

The volume of a cuboid is $V = l \times w \times h$. So the volumes are

Cuboid	**Volume**
Cuboid 1	$1 \times 1 \times 16 = 16$ units cubed
Cuboid 2	$1 \times 2 \times 8 = 16$ units cubed
Cuboid 3	$1 \times 4 \times 4 = 16$ units cubed
Cuboid 4	$2 \times 2 \times 4 = 16$ units cubed

So all the cuboids have the same volume. They are all made up of 16 cubic units.

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How many of cuboid 4 will make a cuboid of dimensions $4 \times 4 \times 4$? Use words, diagrams and/or numbers to explain how you know.

Cuboid 4 has dimensions $2 \times 2 \times 4$.

If we stack a copy of cuboid 4 on top of the original one, we'll have a cuboid of size $2 \times 4 \times 4$.

We need to double this again (multiply by 2) to have a cuboid of size $4 \times 4 \times 4$.

So we place two more copies of cuboid 4 beside the two we currently have.

So 4 cuboids of size $2 \times 2 \times 4$ will make a cube of size $4 \times 4 \times 4$.

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You can only make 6 different cuboids with 24 cubes. Complete the table showing the dimensions of the 6 different cuboids.

We need three numbers that multiply to give exactly 24. We can start with $1 \times 1 \times 24$.

We can find more of the combinations by splitting 24 into its factors, $24 = 4 \times 6$ or $24 = 3 \times 8$ or $24 = 2 \times 12$. We can use these with the factor 1. This gives us $1 \times 2 \times 12$ and $1 \times 4 \times 6$ and $1 \times 3 \times 8$.

We can also split 12 into its factors, $12 = 3 \times 4$ or $12 = 2 \times 6$ and multiply the result by 2 to create a cuboid of volume 24. Here's the completed table:

	Dimensions		
Cuboid 1	1	1	24
Cuboid 2	1	2	12
Cuboid 3	1	4	6
Cuboid 4	1	3	8
Cuboid 5	2	3	4
Cuboid 6	2	2	6

Question 22

Four-fifths of the members of a club are female. Three-quarters of these females are over 20 years old. What fraction of the members of the club are females over 20 years old?

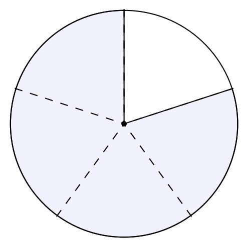
Use words, diagrams and numbers to explain how you know.

We know that $\frac{4}{5}$ of the club members are female. Of these members $\frac{3}{4}$ are over 20 which means we need to find $\frac{3}{4}$ of $\frac{4}{5}$ which we do by multiplying:

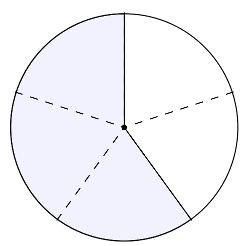
$$\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}$$

So $\frac{3}{5}$ of the club members are females over 20 years old.

We can use a diagram to represent $\frac{4}{5}$ as follows:



Now we can see the highlighted area is split into into four equal pieces. Since we need to find $\frac{3}{4}$ of this area, we count three of the four highlighted segments to get:



This area is $\frac{3}{5}$ of the club members.

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Question 23

The table shows a recipe for a fruit drink.

Type of Juice	Amount
Orange	$\frac{1}{2}$ litre
Apple	$\frac{1}{3}$ litre
Blackcurrant	$\frac{1}{6}$ litre
Total	**1 litre**

I want to make $1\frac{1}{2}$ litres of the same drink.
Complete the table below to show how much of each type of juice to use. Show your working.

We need $1\frac{1}{2}$ times more ingredients. So we multiply by $1\frac{1}{2} = \frac{3}{2}$.

Type of Juice	Amount
Orange	$\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$ litre
Apple	$\frac{1}{3} \times \frac{3}{2} = \frac{3}{6} = \frac{1}{2}$ litre
Blackcurrant	$\frac{1}{6} \times \frac{3}{2} = \frac{3}{12} = \frac{1}{4}$ litre
Total	**1.5 litre**

Use words, diagrams and /or numbers to explain how you know.

$1\frac{1}{2}$ written as an improper fraction is $\frac{2}{2} + \frac{1}{2} = \frac{3}{2}$. Once it is written in this form we can multiply all the ingredients by $\frac{3}{2}$ to calculate the amount of each ingredient needed. We can double check our answer by adding the amount of ingredients to see that

$$\frac{3}{4} + \frac{1}{2} + \frac{1}{4}$$
$$= \frac{3}{4} + \frac{2}{4} + \frac{1}{4}$$
$$= \frac{6}{4}$$
$$= \frac{3}{2} \text{ litres}$$

Question 24

I start with any two consecutive integers. I square each of them, then I add the two squares together.

Use words, letters, diagrams and/or numbers to prove that the total must be an odd number.

Let's start with some examples of square numbers: $2^2 = 4, 3^2 = 9, 7^2 = 49, 10^2 = 100$. From this it looks like there's a difference between odd numbers O squared and even numbers E squared. Odd squared is always odd since it is the same as multiplying two odd numbers. Even squared is always even.

$$O^2 = O$$
$$E^2 = E$$

If I have two square numbers and add them, I could be adding two even numbers, two odd numbers or one odd and one even.

However, we also know that the original integers are consecutive e.g. $2, 3$ or $17, 18$ which means one of them is even and one of them is odd. So we square them first as follows:

$$O^2 + E^2 = O + E$$

since the odd number squared is odd and the even number squared is even. When we add an even and an odd number the result is always odd.

$$O^2 + E^2 = O + E = O$$

Question 25

Think about triangles that have

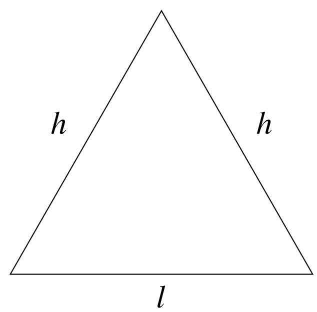
(a) a perimeter of $15cm$,

(b) two or more equal sides,

(c) each side a whole number of centimetres

Prove that there are only **four** of these triangles. You do not need to construct the triangles.

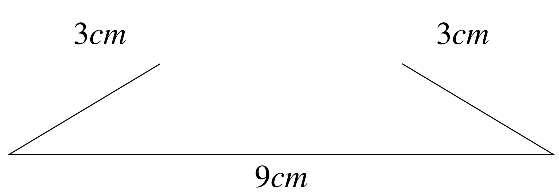
We'll label the equal sides h and the third side l:



Let's list all the numbers for h and l that add to give $15cm$.

Side lengths h, h, l
$1, 1, 13$
$2, 2, 11$
$3, 3, 9$
$4, 4, 7$
$5, 5, 5$
$6, 6, 3$
$7, 7, 1$

Some of these don't form triangles. For example, if the two equal sides were $3cm$, the third side would be $9cm$ since the perimeter would then be $3 + 3 + 9 = 15cm$. But the two equal sides are too short to meet at the top, so this doesn't form a triangle at all.



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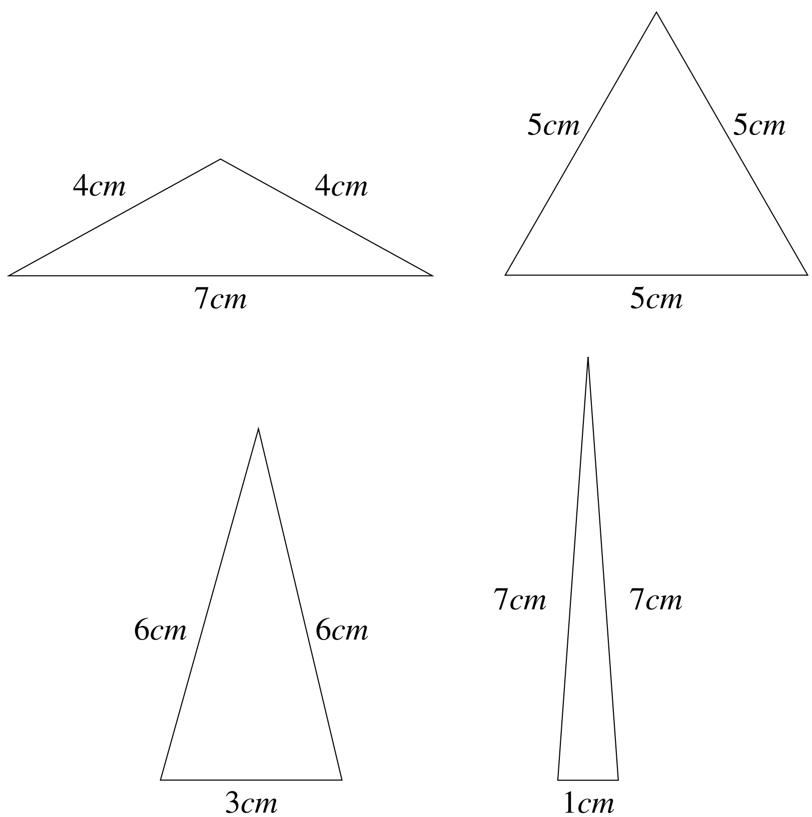
In general, this means that the side l can't be bigger than $h + h = 2h$. So we can exclude all the combinations where $l > 2h$. This leaves four possible triangles.

Side lengths h, h, l	$2h$	l	
1, 1, 13	2	13	$l > 2h$
2, 2, 11	4	11	$l > 2h$
3, 3, 9	6	9	$l > 2h$
4, 4, 7			
5, 5, 5			
6, 6, 3			
7, 7, 1			

Although the question says we do not need to construct these, we can verify our four triangles above with some diagrams:



Question 26

The table shows data about births in Ireland

Year	Number of Births
2002	6.05×10^4
2003	6.15×10^4
2004	6.19×10^4
2005	6.14×10^4
2006	6.54×10^4
2007	7.14×10^4
2008	7.52×10^4
2009	7.56×10^4
2010	7.51×10^4
2011	7.40×10^4
2012	6.17×10^4
2013	6.89×10^4
2014	6.73×10^4
2015	6.55×10^4
2016	6.41×10^4
2017	6.18×10^4
2018	6.10×10^4

In what year was the number of births the highest?

How many more births were there in 2012 than in 2002?

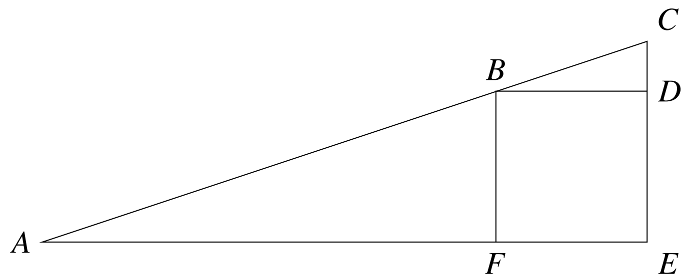
In scientific notation 6.05×10^4 represents 6.05 with the decimal place moved 4 places to the right which is 60500 births. Since all the numbers of births are multiplied by 10^4 we can compare the numbers in front to select the highest number of births. This was 7.56×10^4 which is 75600 births.

In 2012 there were $6.17 \times 10^4 = 61700$ births.
In 2002 there were $6.05 \times 10^4 = 60500$ births.

Subtract the second number from the first to see that there were 1200 more births in 2012 than in 2002.

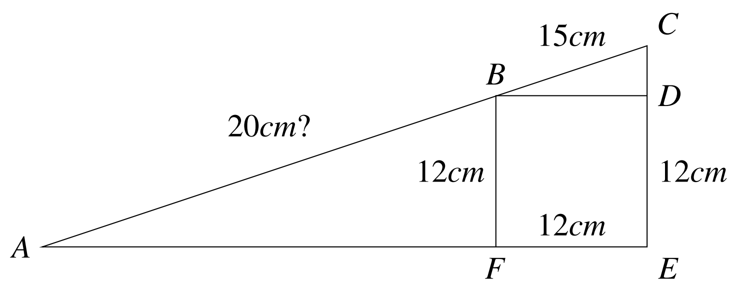
Question 27

The diagram shows a square inside a triangle.



ABC is a straight line. The side length of square *BDEF* is 12*cm*. The length of *BC* is 15*cm*. Show that the length of *AB* is 20*cm*.

Firstly, let's fill in the information:



Since *BD* is 12cm we can work out *CD* using the Pythagorean theorem:

$$12^2 + |CD|^2 = 15^2$$
$$144 + |CD|^2 = 225$$
$$|CD|^2 = 225 - 144$$
$$|CD|^2 = 81$$
$$|CD| = 9$$

The triangles *ABF* and *BCD* are similar triangles since they both have a right angle, and $\angle BAF = \angle CBD$ because *ABC* is a straight line and *BF* and *CE* are parallel. The sides of triangles *ABF* and *BCD* are in the same ratio. To figure out this ratio compare the short sides *BF* and *CD* of the two. They are in the ratio $\frac{12}{9} = \frac{4}{3}$ which means the sides of the big triangle are $\frac{4}{3}$ times bigger than the sides of the smaller one.

This means the the long side $|AB| = \frac{4}{3} \times |BC| = \frac{4}{3} \times 15 = 20cm$.