Question 1 **(Suggested maximum time: 5 minutes)**

(a) **(i)** Write the numbers 3, 9 and 25 into the three empty boxes below to make the mathematical statement true. Use each number only once.

$$\frac{3}{5} + \frac{9}{25} = \frac{24}{25}$$

We can use a common denominator to check that

$$\frac{3}{5} + \frac{9}{25} = \frac{3(5) + 9(1)}{25} = \frac{15 + 9}{25} = \frac{24}{25}$$

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(ii) Write the numbers 3, 5, 9 and 25 into the empty boxes below so that the **difference** between the two fractions is as large as possible. Use each number only once.

$$\frac{25}{3} - \frac{5}{9}$$

To get the biggest difference, we want to create the smallest number we can, and take it away from the biggest number we can create.

First we'll create the largest fraction in the first term by putting the largest number, 25, in the numerator and the smallest number, 3, in the denominator.

Next, we want to make the negative term as small as possible, so put the largest remaining number, 9, in the denominator and the final number, 5, in the numerator.

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(b) A positive whole number has exactly 4 factors. One of the factors is 9. Work out the number.

Let's call this number $9x$, as we know it has a factor of 9. This number has three known factors, 1,3 and 9, and one unknown factor, x.

If we let $x = 2$, we would result in more than one new factor since $3^2 \times 2 = 18$ which has 6 factors $1, 2, 3, 6, 9$ and 18.

If $x = 3$ then it results in only one new factor i.e. $3^2 \times 3 = 27$ which has 4 factors $1, 3, 9$ and 27.

The number we're looking for is 27, which has exactly four factors, one of which is 9, as required.

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Question 2 **(Suggested maximum time: 10 minutes)**

Millie bakes cakes and sells them at the local market.

(a) Millie needs 4 eggs to make each cake. She has 28 eggs.
How many cakes can she make?

Since it takes 4 eggs to make each cake we can divide the total of eggs by 4 to see that Millie can make $28 \div 4 = 7$ cakes.

(b) Millie makes a filling for her cakes using only butter and sugar.
The ratio of the weight of butter to sugar is 5 : 7.

One day, Millie makes a total of 2.4 kg of filling.
Work out how many **grams** of **sugar** Millie used to make this filling.

The ratio 5 : 7 contains $5 + 7 = 12$ parts which are split between butter and sugar. We should divide the total amount of filling into 12 parts: 2.4 kg $\div 12 = 0.2$ kg or 200 grams per part.

The amount of butter is 5 parts which is 5×200 g = 1000 g.
The amount of sugar is 7 parts which is 7×200 g = 1400 g.

This gives 1000 g of butter and 1400 g of sugar which adds up to 2400 g or 2.4 kg of ingredients.

(c) Millie is buying flour at her local shop. The shop has two special offers:

Special Offer A	Special Offer B
1 kg bags: €3.50 each Special offer: 3 bags for the price of 2	1.5 kg bags: €5 each Special offer: 20% off

Millie wants to buy 6 kg of flour. Work out which offer, **A** or **B**, will give her the better value.

Better value: (tick (✓) **one** box only)	**Offer A** ☑	**Offer B** ☐

Special Offer A: Since these are 1 kg bags, Millie must buy 6 bags since 6×1 kg = 6 kg. If she buys 6 bags she gets these for the price of 4 bags which costs $4 \times €3.50 = €14$.

Special offer A costs €14.

Special Offer B: Since these are 1.5 kg bags, Millie must buy 4 bags since 4×1.5 kg = 6 kg. If Millie buys 4 bags at a cost of $4 \times €5 = €20$ she gets a 20% discount which is

$$€20 \times 20\% = €20 \times \frac{20}{100} = €4.$$

Millie pays €20 minus the discount of €4.

Special offer B costs €16. Special offer A is cheaper.

(d) Millie sells each cake for €7.50.
This gives her a profit of 20%.
Work out how much it **costs** Millie to make each cake.

The selling price of €7.50 includes Millie's 20% profit which means the selling price €7.50 = 120% of the cost price. We want to find 100% of the cost price.

$$€7.50 = 120\%$$
$$€0.0625 = 1\%$$
$$€6.25 = 100\%$$

The cost price is €6.25.

(e) Millie has €3000 in a special savings account.
It has an interest rate of 2.5% per year for 4 years, compounded annually.
She does not put any money in or take any money out of the account over the 4 years.

Work out the **total** amount in the account after the 4 years.
Give your answer correct to the nearest cent.

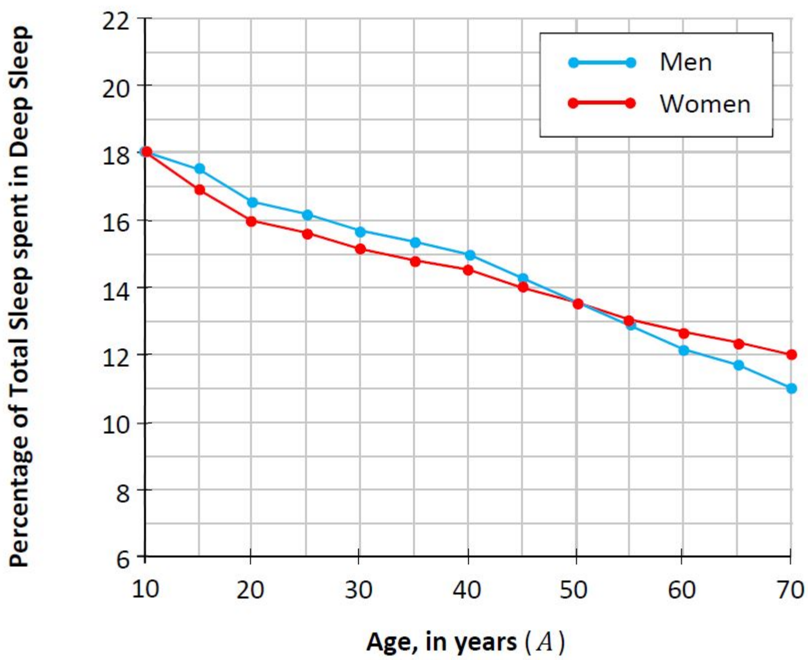
The interest rate $i = \frac{2.5}{100}$, the present value is €3000, the time invested is $t = 4$ years. The compound interest formula says that the final value F is

$$F = 3000\left(1 + \frac{2.5}{100}\right)^4 = 3000(1.025)^4 = €3311.439$$

The total in the account after 4 years is €3311.44 correct to the nearest cent.

Question 3 **(Suggested maximum time: 10 minutes)**

The diagram below shows the percentage of their total sleep that men and women spend in deep sleep, depending on their age in years, A.



Gina is a 20-year-old woman.

(a) Use the diagram to estimate the percentage of her total sleep that Gina spends in deep sleep. Show your work on the graph.

Since Gina is 20 years old, we should look above 20 on the horizontal axis. She is a woman so we look at the red line which shows 16% deep sleep for women aged 20.

From the graph, we estimate Gina spends 16% of her total sleep in deep sleep.

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(b) Gina sleeps an average of 8 hours in total each night.
Work out how many hours Gina spends in deep sleep on average **each week** (7 nights).

In one night Gina sleeps 8 hours and spends 16% of that time in deep sleep which is a total of $16\% \times 8$ hours $= \frac{16}{100} \times 8 = 1.28$ hours of deep sleep per night.

In one week she gets 7 nights sleep which is a total of $7 \times 1.28 = 8.96$ hours of deep sleep per week.

We estimate that Gina spends 8.96 hours per week in deep sleep.

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(c) Use the diagram to fill in the inequality in A below to show the age range for which **women** spend a **lower** percentage of their sleep in deep sleep than men do.

$$10 < A < 50$$

(d) The data in the survey was collected from 6 billion nights of sleep, where a billion is a thousand million. Write 6 billion in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{N}$.

The significant digit in 6 billion is 6. We know a billion is a thousand million. We write those numbers in scientific form as follows:

$1000 = 10^3$ a thousand

$1000000 = 10^6$ a million

This means that a billion is $10^3 \times 10^6 = 10^9$. Thus

$$6 \text{ billion} = 6 \times 10^9.$$

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Phillippe uses a **linear** model to estimate the percentage of their total sleep that men spend in deep sleep, from 42 to 66 years of age. Some of his results are in the table below.

(e) Complete the table below so that the percentages follow a linear pattern. Show your working out.

Age, A (years)	40	45	50	55	60
Percentage of Total Sleep spent in Deep Sleep	15.0	14.3	13.6	12.9	12.2

Between ages of 40 and 50 the percentage changes from 15.0% to 13.6% which is a decrease of 1.4 percent. Between 50 and 60 the percentage decreases by the same amount.

Since it is a linear model, the change between the ages of 40 and 45 is a decrease of 0.7, half of the change between 40 and 50 years.

We can fill out the table as follows:

$$15.0 - 0.7 = 14.3$$
$$14.3 - 0.7 = 13.6$$
$$13.6 - 0.7 = 12.9$$
$$12.9 - 0.7 = 12.2$$

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(f) Using the values in the table above, Phillippe writes the percentage of sleep spent in deep sleep as a function of age, A, for $40 \leq A \leq 60$. The graph of this function is a line with a slope of -0.14.

Explain what -0.14 means in the context of this question.

The slope of the line is -0.14. This means that a one unit increase in age, results in a -0.14 change in percentage.

Every 1 year increase in age decreases the percentage of total sleep spent in deep sleep by 0.14%.

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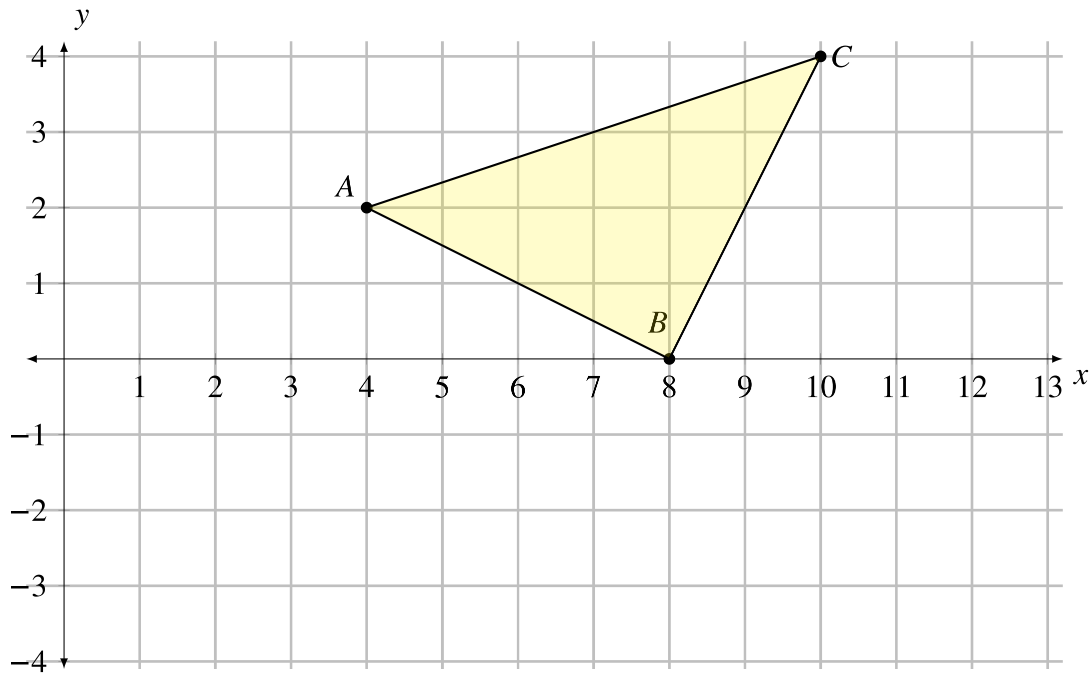
Question 4 **(Suggested maximum time: 15 minutes)**

The co-ordinate diagram below shows the triangle ABC.
The point A has co-ordinates $(4, 2)$.



(a) Write down the co-ordinates of the point B and the point C.

$B =$ (8 , 0) $C =$ (10 , 4)

(b) The table below shows the equations of the lines AC and AB, where $m, k \in \mathbb{Q}$.
Work out the value of m and the value of k.

Line AC	**Line** AB
Equation: $y = mx + \frac{2}{3}$	Equation: $y = -\frac{1}{2}x + k$
Answer: $m = \frac{1}{3}$	Answer: $k = 4$

The line AC goes through the point $C = (10, 4)$. Thus

$$\begin{aligned} 4 &= m(10) + \frac{2}{3} \\ 4 - \frac{2}{3} &= 10m \\ \frac{10}{3} &= 10m \\ \frac{1}{3} &= m \end{aligned}$$

The line AB goes through the point $B = (8, 0)$. Thus

$$\begin{aligned} 0 &= -\frac{1}{2}(8) + k \\ 0 &= -4 + k \\ 4 &= k \end{aligned}$$

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(c) Show that the **area** of the triangle ABC is 10 square units.

It looks like the angle $\angle ABC$ is a right-angle. We can check by finding the slope of the line BC. We have two points $B = (8,0) = (x_1, y_1)$ and $C = (10,4) = (x_2, y_2)$. The slope is

$$\begin{aligned} \text{Slope } BC &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-0}{10-8} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

We know the slope of the line AB is $-\frac{1}{2}$ from the previous part. Since $-\frac{1}{2} \times 2 = -1$ we know that AB is perpendicular to BC and therefore $\angle ABC = 90^\circ$.

Let's consider the length $|BC|$ to be the base of this right-angled triangle. Then perpendicular height is the length $|AB|$. We can use the distance formula to find these lengths:

$$\begin{aligned} |AB| &= \sqrt{(8-4)^2 + (0-2)^2} \\ &= \sqrt{4^2 + 2^2} \\ &= \sqrt{16+4} \\ &= \sqrt{20} \end{aligned}$$

$$\begin{aligned} |BC| &= \sqrt{(10-8)^2 + (4-0)^2} \\ &= \sqrt{2^2 + 4^2} \\ &= \sqrt{14+6} \\ &= \sqrt{20} \end{aligned}$$

So the area of the triangle $\triangle ABC$ is

$$A = \frac{1}{2} \times b \times h = \frac{1}{2} \times \sqrt{20} \times \sqrt{20} = \frac{1}{2} \times 20 = 10 \text{ units squared.}$$

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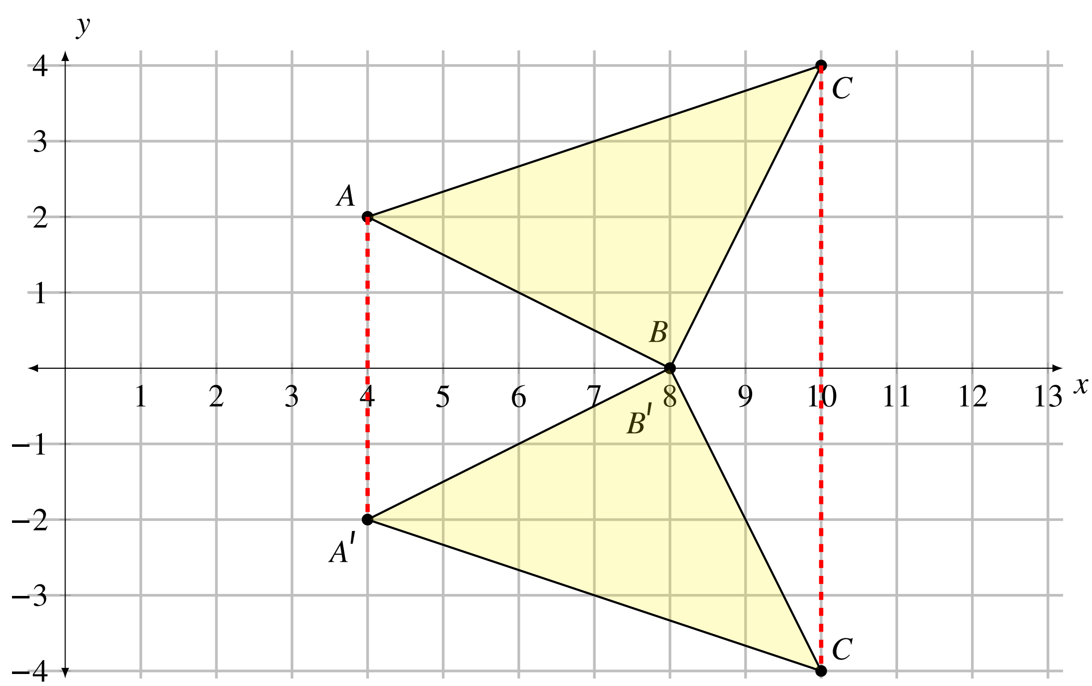
The triangle $A'B'C'$ is the image of ABC under **axial symmetry** in the x axis.

(d) **Draw** the triangle $A'B'C'$ on the co-ordinate diagram.

Axial symmetry in the x-axis means the y-values will be changed by a factor of -1 so the new points are:

$$A' = (4, -2)$$
$$B' = (8, 0)$$
$$C' = (10, -4)$$

We can also do this graphically by drawing perpendicular lines to the x-axis and continuing them by the same distance on the other side.



(e) Which of the points A, B, or C is in the **intersection** of the two triangles ABC and $A'B'C'$?

Answer = $B = (8, 0)$

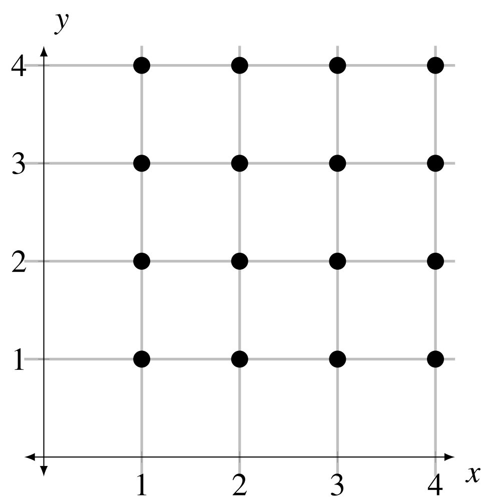
(f) The point (p, s) lies **inside** the triangle ABC, where $p, s \in \mathbb{R}$.
Use this fact to write down the co-ordinates of a point that **must** lie **inside** the triangle $A'B'C'$, giving your answer in terms of p and s.

Answer = $(p, -s)$

Axial symmetry in the x-axis changes the y coordinates by a factor of -1.

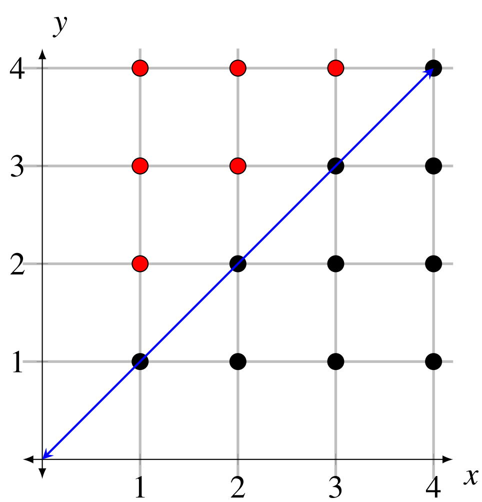
Question 5 **(Suggested maximum time: 5 minutes)**

In the co-ordinate diagram below, 16 points are marked with a dot (•).



(a) Louise picks 1 point at random from the 16 points marked with a dot in the diagram. She then finds the equation of the line that goes through this point and through (0, 0).
Find the **probability** that Louise's line has a slope that is **greater than** 1.

The line that goes through the origin and the points $(1,1)$, $(2,2)$, $(3,3)$ and $(4,4)$ has slope 1. To have a slope **greater than** 1, Louise must pick one of the 6 red points out of the total 16 points.



The probability of Louise randomly picking a line of slope greater than 1 is $\frac{6}{16} = \frac{3}{8}$.

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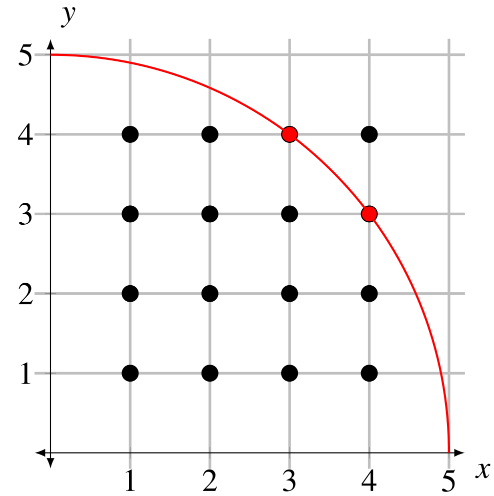
(b) How many of the 16 points marked with a dot in the diagram are a distance of **exactly 5 units** from the point (0, 0)?

Using your compass draw an arc of radius 5 centred at the origin $(0,0)$:



We can see that the points $(3,4)$ and $(4,3)$ are exactly 5 units from the origin.

Let's verify the distance between $O = (0,0)$ and the points $(3,4)$ and $(4,3)$:

$$\sqrt{(3-0)^2+(4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$$
$$\sqrt{(4-0)^2+(3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

This confirms there are exactly two of the 16 points which are exactly 5 units from the origin.

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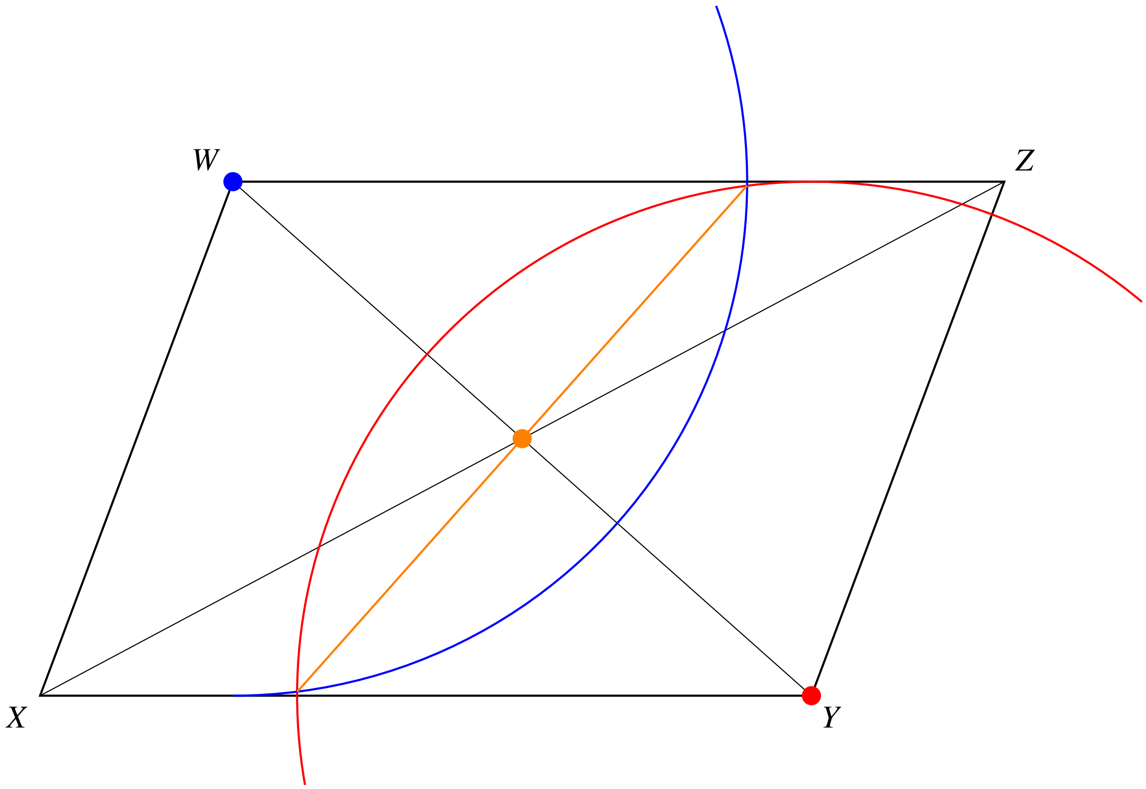
Question 6 **(Suggested maximum time: 10 minutes)**

(a) The diagram below shows a quadrilateral $WXYZ$, as well as its diagonals $[WY]$ and XZ.

Without measuring, perform **constructions** on the diagram below to show that the diagonals **bisect** each other. Show any necessary construction lines clearly.



First draw an arc centred at the point Y whose radius is more than half the distance $|YW|$. Then draw an arc centred at W with the same radius.

Join the two points of intersection. The point where this intersects the line YW is the bisection point of YW.

Perform a similar construction to bisect XZ. This will also be the point of intersection of the two diagonals. We've left this out for clarity. Please complete it as an exercise.

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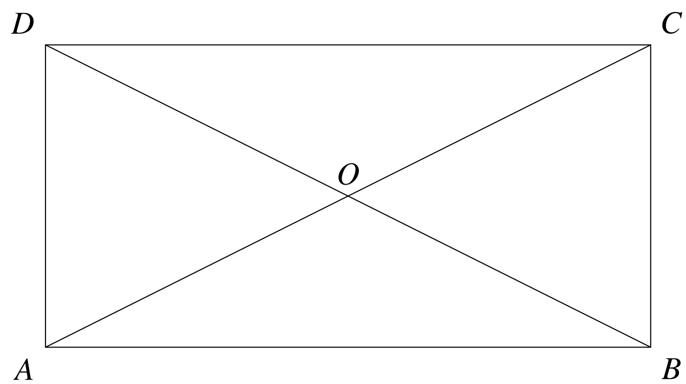
This question continues on the next page...

(b) The diagram below shows the parallelogram $ABCD$. Its diagonals meet at the point O. In this parallelogram, $|\angle DAB| = |\angle ABC|$.

Prove that the triangle ABO is **isosceles**.
Give a reason for each statement that you make in your proof.

Hint: it may help to show that the triangles DAB and CBA are congruent.



In a parallelogram, opposite angles are equal, so we know that:

$$\angle DAB = \angle BCD \qquad \text{and} \qquad \angle ABC = \angle ADC$$

We also know that all four angles add up to 360°. We can use the equations above to show that:

$$\angle DAB + \angle BCD + \angle ABC + \angle ADC = 360^\circ$$
$$\angle DAB + \angle DAB + \angle ABC + \angle ABC = 360^\circ$$
$$2(\angle DAB + \angle ABC) = 360^\circ$$

The question says that in this parallelogram $\angle DAB = \angle ABC$ so we get:

$$2(\angle DAB + \angle ABC) = 360^\circ$$
$$2(\angle DAB + \angle DAB) = 360^\circ$$
$$4(\angle DAB) = 360^\circ$$
$$\angle DAB = 90^\circ$$

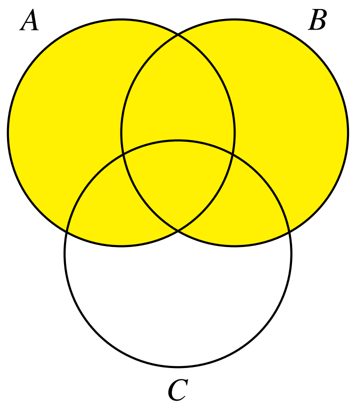
This means that all angles in this parallelogram must be 90° and therefore this shape is a rectangle. Let's compare the triangles $\triangle DAB$ and $\triangle CBA$.

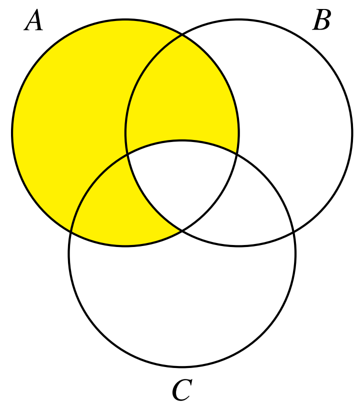
- They both have a 90° angle
- They have the side AB in common.
- The sides AD and CD are equal since $ABCD$ is a parallelogram

Therefore by SAS (side-angle-side) these triangles are congruent. This means that their corresponding angles $\angle CAB$ and $\angle DBA$ are equal. Therefore $\triangle AOB$ must be an isosceles triangle.

Question 7 **(Suggested maximum time: 5 minutes)**

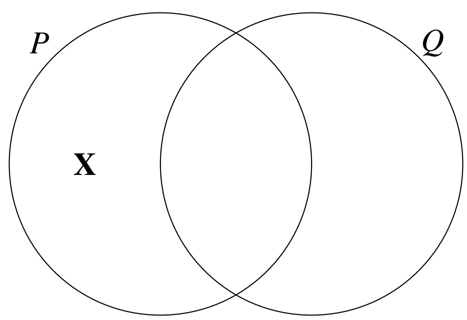
(a) A, B, and C are sets.
Complete the tables below by **shading in** each of the given sets in the Venn diagram.

Set	$A \cup B$
Venn Diagram	

Set	$A \setminus (B \cap C)$
Venn Diagram	

(b) P and Q are two other sets. P is a **subset** of Q.

(i) Write an **X** in the region of the Venn diagram below which must contain **no** elements.



(ii) Put a tick (✓) in the correct box to show which statement **must** be true.
Tick **one** box only. Explain your answer.

$\#P \leq \#Q$ ☑ $\#P = \#Q$ ☐ $\#P \geq \#Q$ ☐

We know that P is a subset of Q. That means that every element of P is in the set Q. Therefore Q must contain at least as many elements as P does.

So the first statement $\#P \leq \#Q$ must be true.

$\#P = \#Q$ is not necessarily true because Q could have more elements than P.

$\#P \geq \#Q$ can not be true.

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Question 8 **(Suggested maximum time: 10 minutes)**

An **equilateral** triangle XYZ has sides of length 6 cm.

(a) Write down the size of the angle $\angle XYZ$.

In an equilateral triangle, each angle is 60°. So $\angle XYZ = 60^\circ$.

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(b) Work out the length of the **perimeter** of the triangle XYZ.

Since the triangle has three sides of length 6 cm, the perimeter is $6 + 6 + 6 = 18$ cm.

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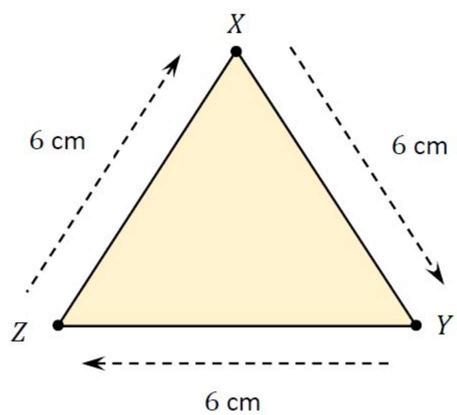
(c) Maria plays a game using the triangle XZY. She flips a coin and moves the counter in the direction XYZ, as shown in the diagram below, using the following rule:

- if she gets **heads**, she moves the counter 6 cm
- if she gets **tails**, she moves the counter 12 cm.



At the start of the game, the counter is at the point X.

Maria gets **tails** on each of the first 4 **flips** of the coin.
Which point is the counter at after the first 4 flips of the coin?

When Maria gets tails on the first flip she moves the counter from X a distance of 12 cm to position Z. Let's do a table to follow the four flips.

Flip number	1	2	3	4
Result	tails	tails	tails	tails
Distance moved	12 cm	12 cm	12 cm	12 cm
New position	Z	Y	X	Z

So the counter is at point Z after the first 4 flips.

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(d) Jodie makes a solid shape using equilateral triangles as faces.
It has E edges and F faces, where $E, F \in \mathbb{N}$. For Jodie's shape:

$$\frac{8F}{5} - E = 2$$
$$3F = 2E$$

Solve these simultaneous equations to find the value of E and the value of F.

$$\frac{8F}{5} - E = 2 \qquad (1)$$
$$3F - 2E = 0 \qquad \text{(re-arranged)} \qquad (2)$$

Multiply the first equation by 5 to eliminate the fractions:

$$8F - 5E = 10$$
$$3F - 2E = 0$$

Now multiply the first equation by 2 and the second equation by -5 so the coefficents of E will match:

$$16F - 10E = 20$$
$$-15F + 10E = 0$$

Now add the equations to get

$$16F - 15F = 20$$
$$F = 20$$

Substitute $F = 20$ into equation (2) above to get:

$$3(20) - 2E = 0$$
$$60 = 2E$$
$$30 = E$$

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Question 9 **(Suggested maximum time: 15 minutes)**

(a) A can in the shape of a cylinder has a radius of 3.6 cm and a height of 10 cm. Work out the **volume** of the can. Give your answer in cm^3, correct to two decimal places.

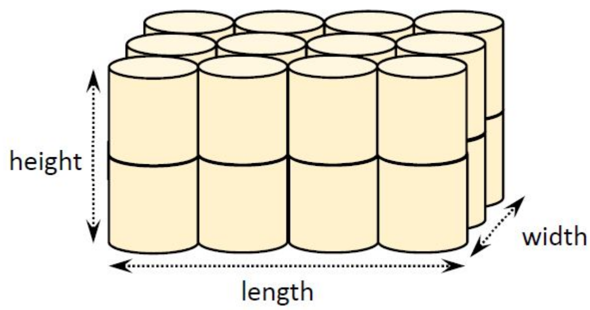
The cylinder has radius $r = 3.6$ and height $h = 10$. So the volume is:

$$V = \pi(3.6)^2(10) = 407.1504 \text{ cm}^3$$

The volume of the cylinder is $V = 407.15 \text{ cm}^3$ correct to two decimal places.

24 of these cans are to be packed into a closed rectangular box.

(b) The cans will be arranged inside the box as follows:



(i) Write down the height, the length, and the width of the smallest rectangular box that will be needed for these 24 cans. One is already done for you.

Height = 20 cm length = 28.8 cm width = 21.6 cm

The height is two cylinders which is $2h = 2(10) = 20$ cm.
The width is 3 cylinders which is $3(2r) = 6r = 6(3.6) = 21.6$ cm.

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(ii) Work out the **volume** of this box.

The volume of the box is the length × width × height which is

$$20 \times 28.8 \times 21.6 = 12441.6 \text{ cm}^3.$$

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(iii) Work out the **percentage** of the volume of this box that is taken up by the 24 cans.
Give your answer correct to one decimal place.

Each cylinder takes up 407.15 cm^3 which means 24 cylinders have a total volume of 9771.6 cm^3.

The box has volume 12441.6 cm^3. So the percentage of the box taken up by the cylinders is

$$\frac{9771.6}{12441.6} \times 100 = 78.539\%$$

The percentage of the box taken up by the cylinders is 78.5% correct to one decimal place.

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There are a number of different ways of arranging the 24 cans so that they can be packed into a rectangular box. The dimensions of the box may be different for different arrangements.

(c) Find the dimensions of the rectangular box required for a **different** arrangement of the 24 cans. Show your working out.

height = 20 cm   length = 14.4 cm   width = 43.2 cm

Suppose we re-arrange the 24 cylinders into a $2 \times 2 \times 6$ formation.

The height is 2 cylinders which is $2h = 20$ cm.
The length is 2 cylinders which is $2(2r) = 4r = 14.4$ cm.
The width is 6 cylinders which is $6(2r) = 12r = 43.2$ cm.

Question 10 **(Suggested maximum time: 5 minutes)**

(a) Write three integers into the following boxes so that the three numbers have:

- a mode of 2
- a mean of 5

Answer: : | 2 |, | 2 |, and | 11 |

The mode is the number that appears most frequently, so we must include 2 twice or more times.

To get a mean of 5 we must use a third number that is not 2.

To work out the last number x given that the mean must be 5:

$$\frac{2+2+x}{3} = 5$$
$$4+x = 15$$
$$x = 11$$

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(b) Write five integers into the following boxes so that the five numbers have:

- a mode of 4
- a median of 4
- a mean of 5
- a range of 12

Answer: : | 1 |, | 3 |, | 4 |, | 4 |, and | 13 |

The number 4 must appear twice or more since it is the mode. The median is the middle value so 4 must be in the middle box. We also know that the largest value is 12 bigger than the smallest value. The integers could look like:

$$x, \; y, \; 4, \; 4, \; x+12$$

where x and y are integers we need to find.

The mean is 5 which means:

$$\begin{aligned} \frac{x+y+4+4+x+12}{5} &= 5 \\ 2x+y+20 &= 25 \\ 2x+y &= 5 \end{aligned}$$

By trial and error we can now choose an integer value of x, and use this to solve the equation for y.

Let $x = 1$, then:

$$\begin{aligned} 2x+y &= 5 \\ 2(1)+y &= 5 \\ 2+y &= 5 \\ y &= 5-2 \\ y &= 3 \end{aligned}$$

When $x = 1$ we have $x+12 = 13$ so the numbers are:

$$1, \; 3, \; 4, \; 4, \; 13$$

You can choose other values of x and calculate y. Always check your solution to see if it matches the four conditions given.

Question 11 **(Suggested maximum time: 5 minutes)**

(a) Find the value of $\dfrac{2p-1}{\sqrt{p^2+15}}$ when $p=-7$.

$$\frac{2(-7)-1}{\sqrt{(-7)^2+15}} = \frac{-14-1}{\sqrt{49+15}}$$
$$= \frac{-15}{\sqrt{64}}$$
$$= -\frac{15}{8}$$

(b) Factorise fully $5fh-2h^2-6h+15f$.

First group the terms that have a common variable factor and factorise:

$$5fh+15f-2h^2-6h = 5f(h+3)-2h(h+3)$$

Now $(h+3)$ is a common factor of these two terms so we can factorise it as:

$$(5f-2h)(h+3)$$

(c) Write the following as a single fraction.

$$\frac{5}{2x+1} - \frac{x}{4}$$

The common denominator is $4(2x+1)$. Then we can re-write the fractions into one as:

$$\begin{aligned}\frac{5}{2x+1} - \frac{x}{4} &= \frac{5(4) - x(2x+1)}{4(2x+1)} \\ &= \frac{20 - 2x^2 - x}{8x+4} \\ &= \frac{-2x^2 - x + 20}{8x+4}\end{aligned}$$

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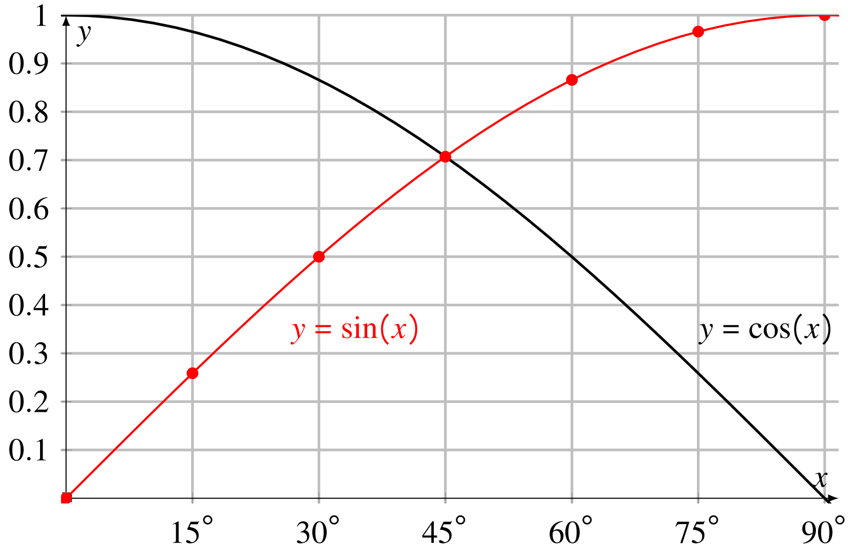
Question 12 **(Suggested maximum time: 5 minutes)**

The co-ordinate diagram below shows the graph of the function $y = \cos(x)$ for $0^\circ \leq x \leq 90^\circ$.



(a) Use a calculator to work out the value of $\sin(60^\circ)$, correct to one decimal place.

Make sure your calculator is set for degree mode (DEG).

$$\sin(60^\circ) = 0.866$$

So $\sin(60^\circ) = 0.9$ correct to one decimal place.

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(b) **Draw** the graph of $y = \sin(x)$ on the diagram above, using the same axes, scales, and domain. Note that $\sin(0^\circ) = 0$ and $\sin(90^\circ) = 1$.

We know that the graph must pass through the points $(0^\circ, 0)$, $(90^\circ, 1)$ and $(60^\circ, 0.9)$ from part (a). Plot some more points by finding sin of each of the x values as follows:

x°	$\sin(x^\circ)$
15°	$\sin(15^\circ) = 0.2588$
30°	$\sin(30^\circ) = 0.5$
45°	$\sin(45^\circ) = 0.707$
75°	$\sin(75^\circ) = 0.966$

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(c) Using your graph from part **(b)**, estimate the **point of intersection** of $y = \cos(x)$ and $y = \sin(x)$, for $0^\circ \le x \le 90^\circ$.

Answer = $(\ 45^\circ\ ,\ 0.7\ )$

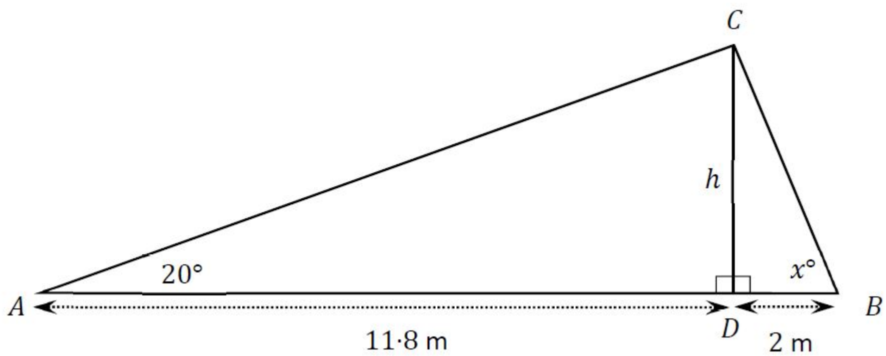
Question 13 **(Suggested maximum time: 5 minutes)**

The photograph on the right shows a house. Part of the roof of this house is shown in the diagram below.

AB is perpendicular to DC.
$|AD| = 11.8$ m and $|DB| = 2$ m.
$|\angle CAD| = 20^\circ$ and $|\angle DBC| = x^\circ$.
Note: $\angle ACB$ is **not** a right angle.

Use **trigonometry** to work out the value of x.
Give your answer correct to the nearest whole number.

Source of image:
www.interiordesign.net. Altered.



Let's find the length h first. We will use tan of the angle 20° since h is the opposite side and the adjacent side has length 11.8 m.

$$\begin{aligned} \tan(20^\circ) &= \frac{h}{11.8} \\ 11.8\ \tan(20^\circ) &= h \\ 4.295 \text{ m} &= h \end{aligned}$$

Now we'll find an expression for $\tan x^\circ$ since the opposite side is $h = 4.295$ m and the adjacent side has length 2 m.

$$\begin{aligned} \tan x^\circ &= \frac{4.295}{2} \\ \tan x^\circ &= 2.1475 \\ x^\circ &= \tan^{-1}(2.1475) \\ x^\circ &= 65.031^\circ \end{aligned}$$

So $x = 65$ correct to the nearest whole number.

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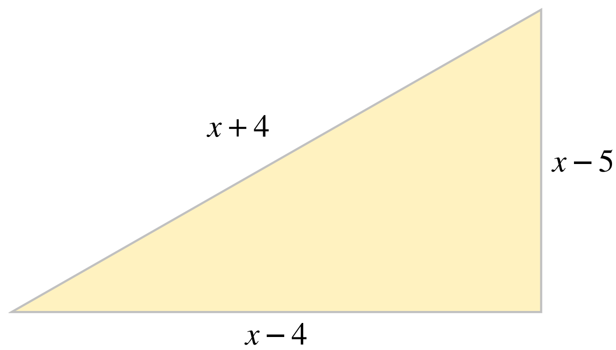
Question 14 **(Suggested maximum time: 5 minutes)**

In the diagram below, the length of each of the sides is given in terms of x, where $x \in \mathbb{N}$.



Show that there is only **one** value of x for which this triangle is right-angled.

Since the triangle is right-angled, we can solve this using Pythagoras' theorem:

$$(x-4)^2 + (x-5)^2 = (x+4)^2$$
$$x^2 - 8x + 16 + x^2 - 10x + 25 = x^2 + 8x + 16$$
$$2x^2 - 18x + 41 = x^2 + 8x + 16$$
$$x^2 - 26x + 25 = 0$$
$$(x-25)(x-1) = 0$$

We solve this equation by letting each factor = 0 and solving for x:

$$x - 25 = 0 \qquad\qquad x - 1 = 0$$
$$x = 25 \qquad\qquad x = 1$$

The two solutions of this equation are $x = 1$ and $x = 25$. However, if $x = 1$ then two of the sides will have negative length, which not possible.

If $x = 25$ the sides lengths will be $29, 20$ and 21 which is a right-angled triangle. So $x = 25$ is the only valid solution. This is the one value of x for which the triangle is right-angled.

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